

# Towards exactness in geomorphometry

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**Abstract** — Exactness of results of geomorphometric research depends not only on measurement and computational exactness but also on exact definition of measured objects and exactness of interpretation of the geomorphometric variables. The need to consider all aspects of exactness in mutual relationships is exemplified from the use of third order local point-based variables.

## I. INTRODUCTION

Geomorphometry is generally considered as one of the most exact parts of geomorphology. This may be why geomorphometry is widely used and developed also in other Earth sciences. DEM quality as well as the preciseness and accuracy of computation of geomorphometric variables are well known factors influencing the exactness of results achieved. However the quality of geomorphometric analysis depends also on exactness in the definition of objects measured, and on unambiguous interpretation of the geomorphometric variables used. Moreover all these aspects are connected and should be considered in mutual relationships (Fig 1).

These aspects are frequently considered independently from each other and interpretation exactness usually receives the least attention. A short overview of the nature and mutual dependence of these aspects is presented next, followed by an outline of the interpretation hierarchy of geomorphometric variables and an example of a comprehensive approach in the use of third order local point-based geomorphometric variables.

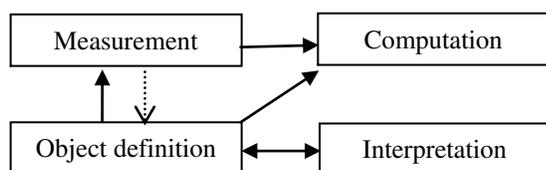


Figure 1. Mutual relationships of main aspects of exactness in geomorphometry

## II. STATE OF THE ART

Although direct measurement of various geomorphometric variables is possible (see e.g. [1]) **measurement exactness** is today nearly exclusively connected with digital elevation model (DEM) creation. Altitudinal precision and grid resolution are the main attributes of quality of the most used grid-based DEMs. The quality of both is, however, created in interaction with other components of geomorphometric exactness. The primary measurements (tacheometry, photogrammetry, radar or lidar) mainly give spatially irregular data; creation of a regular grid by interpolation functions represents a secondary product affected by computation error of the interpolation function. It is clear that the **exactness of object definition** is a major influence on measurement, with knock-on effects on computation and interpretation. The measured objects of geomorphometry (land surface or landforms) are often fuzzy [2]. The land surface is most frequently perceived as the boundary between lithosphere (pedosphere) on the one side and atmosphere or hydrosphere on the other side. But different ways of treating vegetation and various anthropogenic features of the surface lead to different concepts of DEM, DTM (digital terrain model) and DSM (digital surface model), with serious consequences for general geomorphometry.

The situation in specific geomorphometry is even more complex. The problem of definition of geomorphometric individuals is "evergreen" containing mainly the aspects of semantic and spatial definition (e.g. [3], [4]). The proposed hierarchic nature of landforms remains a major problem for object definition exactness. Establishing a nested hierarchy of landforms from a source DEM is one way of dealing with the problem (e.g. [5]). Another way is gradual generalization of DEM, long used in tectonic geomorphology (e.g. the concept of isobase surfaces in the sense of [6], [7]). Very accurate LiDAR measurements have made some generalization of the land surface necessary for morphodynamics (removing small temporary forms such as ploughland, molehills and vehicles). Using of wavelet transform could be a promising approach [8]. Computation of

morphometric variables and derivation of specific geomorphic objects from such generalized land surfaces is important mainly from the aspect of exact morphogenetic interpretation.

**Computational exactness** is generally perceived to result from error in input data and in computational method (e.g. [9], [10]). However the determination of data error depends on the ‘reference standard’ used i.e. the definition of ideal (error-free) land surface. If landforms of higher order are studied, greater detail from more precise data may not be relevant: a generalized land surface should be used as the reference standard, but the myriad possibilities of generalization pose a problem. Moreover the correctness of delimitation of specific landforms determines the quality of computation of all indexes in specific geomorphometry.

As yet, insufficient systematic attention has been paid to **interpretation exactness**. Geomorphometric variables have not only geometrical meaning but also physical (morphodynamic and morphogenetic) meaning. While physical interpretation of simple variables can be relatively clear, interpretation of more complex variables and results (e.g. objects created by various segmentation procedures) is frequently obscure. A large part of the interpretation exactness results from relationships between basic geomorphological categories (Fig. 2) and the complexity of variables. The exactness of correspondence between geometry and its physical interpretation is fundamental. The most straightforward is the physical interpretation of dimensions and positional characteristics of geomorphic objects (Table I). Some derivatives and integrals of them bring greater interpretation risks Curvatures (see [10] for overview and terminology) can be an example. Profile curvature and normal change of slope gradient are physically clear (change of downslope gravity force component or ratio of gravity force components).

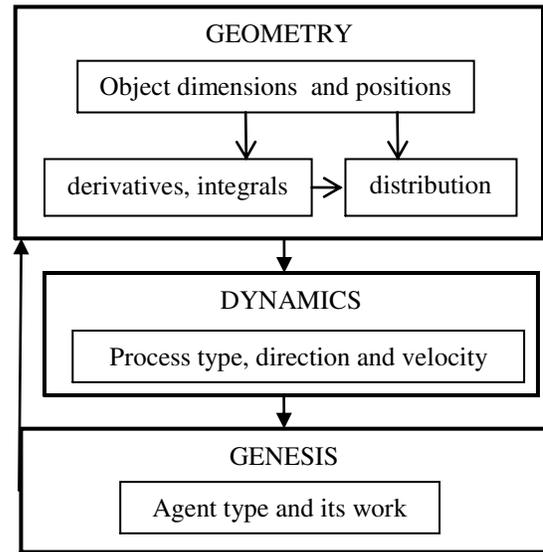


Figure 2. Relationships between basic geomorphological categories

Normal change of slope angle correlates also with acceleration of gravity flows, but precise interpretation is less clear. Similarly, all types of plan curvature reflect concentration/dispersion mechanisms, although the quantitative relations are various and complex. Combined "curvature", newly integrated in ArcGIS, has become increasingly popular despite its lack of clear physical interpretation. Roughness in terms of wavelength and amplitude [11] has a straightforward physical interpretation related to geomorphic work. Analysis of distribution patterns is more complex.

TABLE I. Fundamental dimensional and positional geometric types of geomorphometric variables

Topological dimension of:	0D	1D	2D	3D
<b>Geometric object→ ↓Measuring</b>	<b>POINT</b>	<b>LINE</b>	<b>SURFACE</b>	<b>SOLID</b>
1D (axis z) Distance	<i>Altitude</i> ...	<i>Mean ridgeline height</i> ...	<i>Glock's available relief</i> ...	<i>Cave height</i> ...
2D (axis x,y) Area, Distance in map	<i>Map distance of peaks</i> ...	<i>Map length of thalweg</i> ...	<i>Catchment length and area in map</i> ...	<i>Cave map area and length</i> ...
3D (x,y,z) Volume, Distance and Area in space	<i>Spatial distance of peaks</i> ...	<i>Spatial length of thalweg</i> ...	<i>Catchment length and area in space</i> ...	<i>Cave volume, surface, space length</i> ...

Many indexes created by combination of basic geomorphometric variables have only limited interpretation exactness unless they are based on conceptual models. Physically based indexes (e.g. topographic wetness index - [14]) have interpretation limits resulting from the generalizations used.

### III. THIRD ORDER LOCAL POINT-BASED VARIABLES

The need to use all aspects of geomorphometric exactness can be exemplified for computation of third order local point-based geomorphometric variables - changes of curvatures [13], [10].

The initial target is utilization of curvature change for elementary form definition and delineation. Our concept [4] is

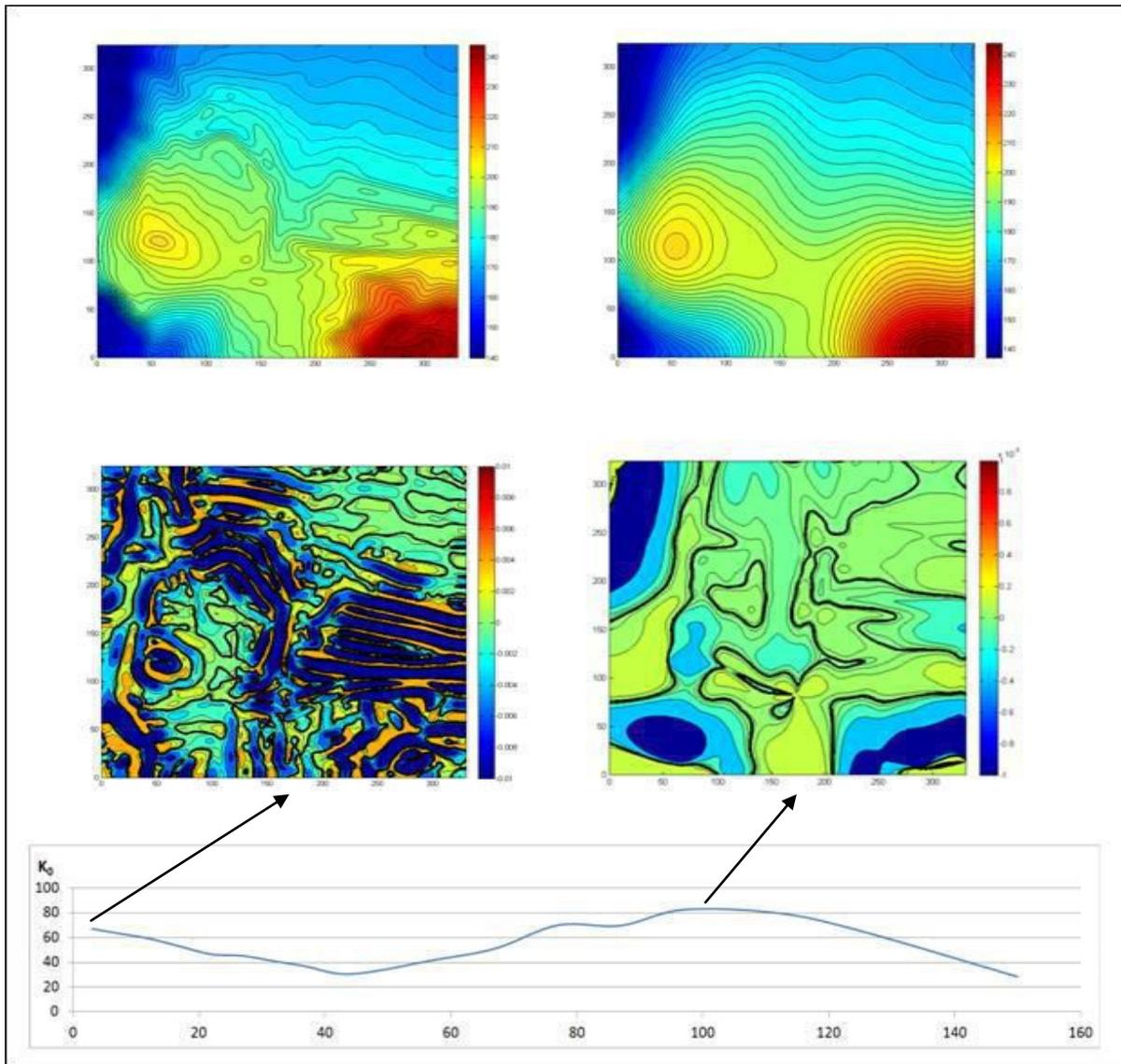


Fig 3. Contourline representation of two hierarchic levels of landforms (top) and corresponding fields of normal change of normal change of gradient -  $G_{nn}$  (centre) detected by maximization of index of concentration of  $G_{nn}$  around zero -  $K_0$  (bottom:  $K_0$  is a function of window size for polynomial models of 6th order.)

based on postulating the existence of dynamic equilibria of land elements in terms of constant values of altitude and some its derivatives (including 3<sup>rd</sup> order). Because it is necessary to eliminate ephemeral landforms, the most precise DEMs have to be generalized to a level where the sought (dynamically stable) elementary forms are evident.

The basic analysis of exactness of five methods of third derivative computation [10] showed varied effectiveness. Most numerical methods that compute a derivative at a chosen grid point are linear functions of values at several nearby grid points. Method error is the difference between the exact derivative of analytical function and the derivative computed using a numerical method applied to data at grid points. Data (DEM) error is the difference between the derivative computed using the chosen numerical method applied to the 'etalon' (exact function values at grid points), and applied to function values at grid points which have some DEM error. We have documented that if for every pair of grid points DEM error is equally distributed and uncorrelated, then the expected value of the second power of data error is smallest for the Least squares method [14] out of all methods available to compute the derivative from the same set of grid points. Extending the number of computational grid points (window) reduces the data error even more but enlarges the method error. On the contrary, raising the order of polynomial reduces the method error but enlarges the data error. While enumeration of change in data error is possible for both cases, enumeration of method error is a complex mathematical problem, so determination of total error is very problematic. Therefore we developed a method generalizing the least square method suggested by [14] for 5 x 5 windows approximated by 3<sup>rd</sup> order polynomials.

Our method enables computation of derivatives for various combinations of window and polynomial order, with subsequent selection of an optimal combination of window and order of polynomial on the basis of a target function. Because of interpretation exactness, instead of profile curvature we use the gradient change and subsequently the normal change of gradient change for target function [4]. The first results suggest that, for a set of differently generalized DEMs, maximisation of a quantile-based measure of kurtosis ( $K_0$ ) of change of gradient change [10] permits selection of the DEM best representing genetically well interpretable landforms of higher order in the streamline direction (Fig. 3).

Geomorphometric variables for a multi-level hierarchy of landforms usually are computed simply by changing input grid resolution [15]. Application of a theoretical assumption into the computational procedure is specific for our approach. Confirmation of the preliminary result on a larger territory could lead not only to a new tool for detection of landforms of various

orders, but also demonstrate the usefulness of incorporating theory when building methodological tools in geomorphometry.

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