Russian Academy of Sciences

SERGEEV INSTITUTE OF ENVIRONMENTAL GEOSCIENCE RAS (IEG RAS)

PROBABILISTIC BEHAVIOR MODELING OF MORPHOMETRIC PARAMETERS FOR THERMOKARST PLAINS WITH FLUVIAL EROSION IN CRYOLITHOZONE

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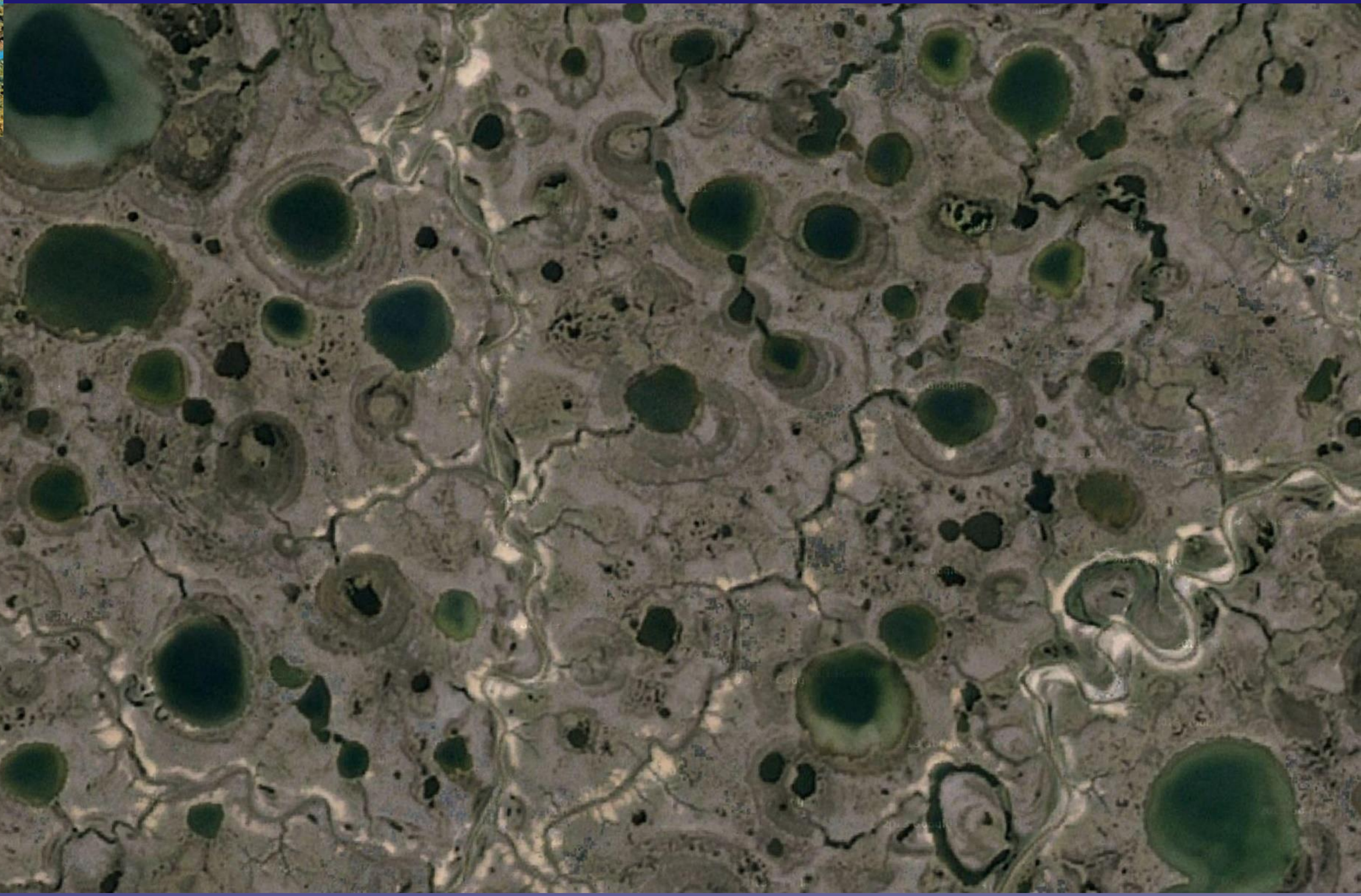
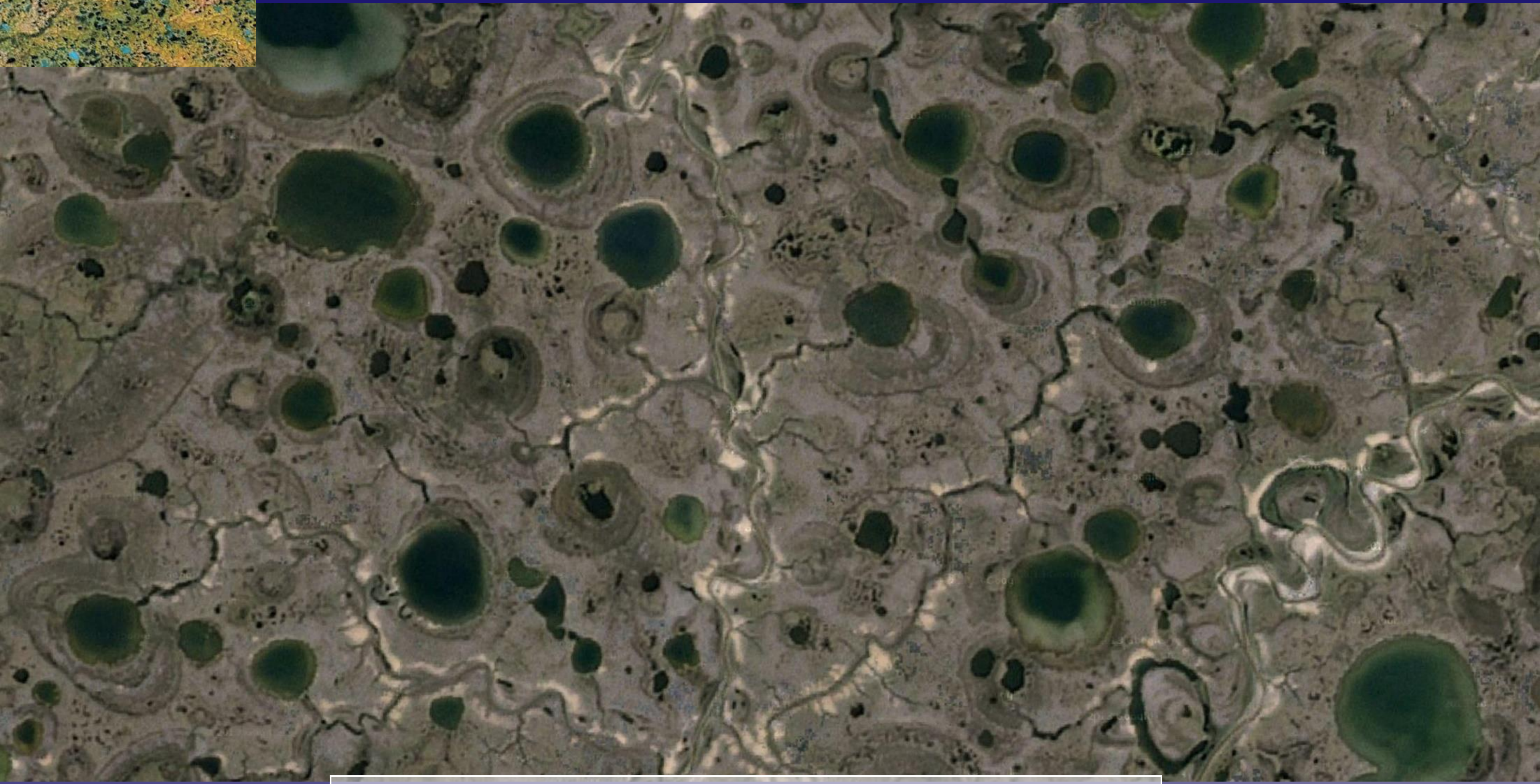
Lacustrine thermokarst plains with fluvial erosion



This research aims to consider the patterns of changing morphometric characteristics of thermokarst plains with fluvial erosion and their interrelations.

Thermokarst plains

Typical morphometrical parameters of the thermokarst plains with fluvial erosion



Location density of lakes,
Location density of khasyreis
Average area of lakes
Average area of khasyreis
Lake area percentage
Location density of fluvial sources

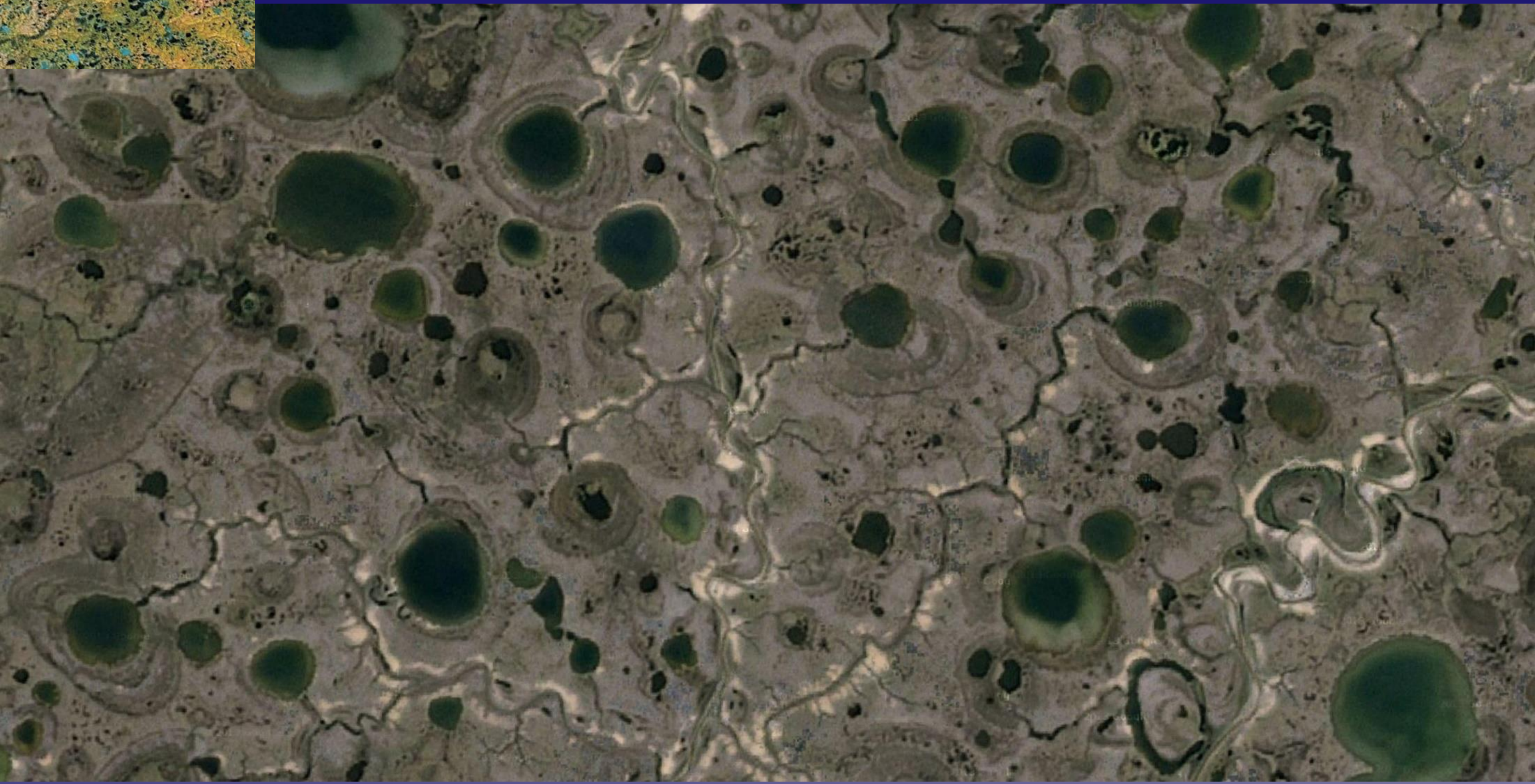
Mathematical model for lacustrine thermokarst plains with fluvial erosion

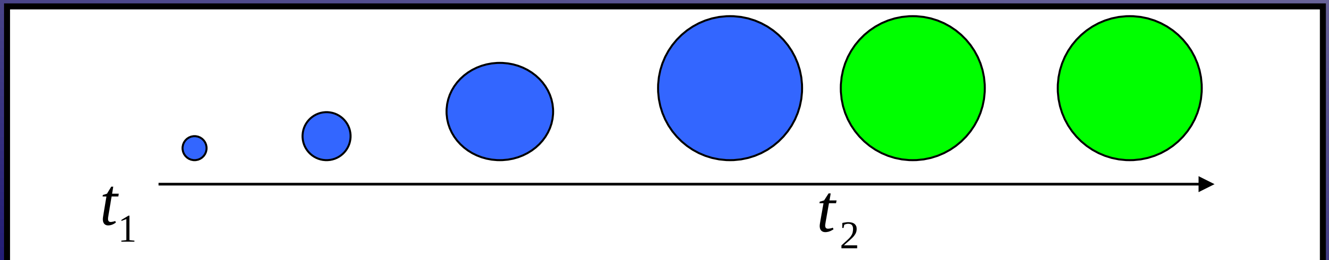
The morphological pattern of these landscapes changes under two opposite processes:

- **appearing and increasing in size of thermokarst lakes,**
- **drainage of the lakes with fluvial erosion and their turn into khasireis.**

Two opposite processes at the same territory is a precondition for a state of dynamic balance.

Mathematical model for lacustrine thermokarst plains with fluvial erosion





Mathematical model for lacustrine thermokarst plains with fluvial erosion.

Model assumptions:

1. Appearance of initial thermokarst depressions (foci) during non-overlapping time intervals (Δt) and non-overlapping sites (Δs) are independent random events. The probability of a depression to appear depends only on the size of the segment and the site

$$p_1 = \lambda \Delta s \Delta t + o(\Delta s \Delta t)$$,

where λ is a parameter.

2. Radius of an appeared thermokarst depression is a random variable being a time function; it is undependable of other lakes and the growth rate is directly proportional to heat storage in the lake water.
3. In the course of its growth, a lake can turn into a khasyrei after draining by the erosion network; probability of this does not depend on development of other lakes. If it happens the depression stops to grow.
4. The appearance of new sources of fluvial erosion within a randomly selected area is a random event and its probability depends only on the area.
5. New lakes do not appear within the already existing thermokarst lakes.

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Mathematical model for lacustrine thermokarst plains with fluvial erosion. Analysis of the model

$$P(k,t)=\frac{[\lambda tS]^k}{k!}e^{-\lambda tS}$$

The number of initial thermokarst depressions within a trial plot

$$f_0(x,t)=\frac{1}{\sqrt{2\pi}\sigma x\sqrt{t}}e^{-\frac{[\ln x-at]^2}{2\sigma^2 t}}$$

Change of lake size (area, diameter) in case of unlimited growth without taking into account possibility of lake drainage

a,σ - parameters, *t* – time, *S* – plot area, λ - generation density of initial thermokarst depressions

Mathematical model for lacustrine thermokarst plains with fluvial erosion. Analysis of the model

$$P(k,t)=\frac{[\eta(t)s]^k}{k!}e^{-\eta(t)s}$$

Distribution of the number of lakes within a random plot at time t

t – time, *S* – plot area,

$\eta(t)$ - average lake location density at time t

Mathematical model for lacustrine thermokarst plains with fluvial erosion. Analysis of the model

$$\lambda_1(t) = \lambda[1 - P_l(t)]$$

An average number of initial thermokarst depressions appearing per a time unit within a trial plot taking into account total lake area percentage

$$F(x) = 1 - e^{-\pi\gamma x^2}$$

the length distribution to the nearest fluvial source

$$f(x,t) = \frac{e^{-\pi\gamma x^2} \int_0^t [1 - P_l(u)] f_0(x, t-u)\, du}{\int_0^t [1 - P_l(u)] \int_0^{+\infty} e^{-\pi\gamma x^2} f_0(x, t-u)\, dx\, du}$$

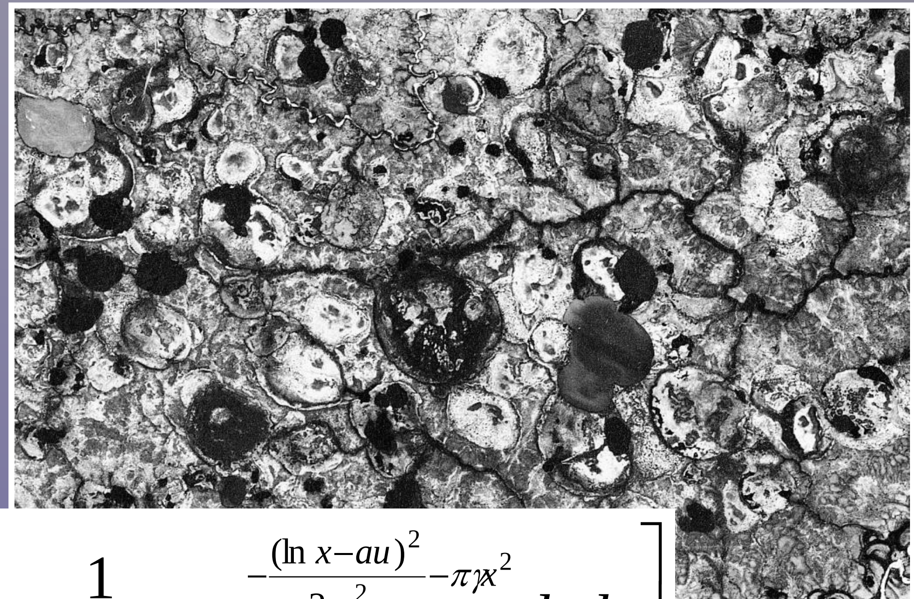
lake radius distribution at moment t

a, σ - parameters, *t* – time, $P_l(t)$ - is a fraction of the whole area occupied by lakes at time t

Mathematical model for lacustrine thermokarst plains with fluvial erosion. Analysis of the model

Dynamics of the lake area percentage

$$P_l(t) = 1 - \exp\left[-\eta(t)\; s_l(t)\right]$$



$$1 - P_l(t) = \exp\left[-\pi\lambda \int_0^t [1 - P_l(u)] \int_0^{+\infty} x^2 \frac{1}{\sqrt{2\pi u}\, x\sigma} e^{-\frac{(\ln x - au)^2}{2\sigma^2 u} - \pi\gamma x^2} dx\,du\right]$$

where

$\gamma, a, \sigma, \lambda$ - parameters, $P_l(t)$ - is a fraction of the whole area occupied by lakes at time t, $s_l(t)$ - an average lake area,

$\eta(t)$ - average lake location density at time t

Mathematical model for lacustrine thermokarst plains with fluvial erosion. Analysis of the model

The main conclusion: after a sufficiently long period of time $t \to +\infty$ since the start of the process a certain stable state appears under a wide spectrum of conditions: a dynamic equilibrium is established between processes of thermokarst lake origination and their turn into khasyreis.

Limit value of thermokarst affect, which is equal to decision of the equation

$$P_l(t) \to P_l^* \;:\; \ln[1-P_l^*] = -\lambda\pi[1-P_l^*]I$$

Limit value for lake location density

$$\eta(\infty) = -\frac{\lambda}{2a} Ei(-\pi\gamma)[1-P_l^*]$$

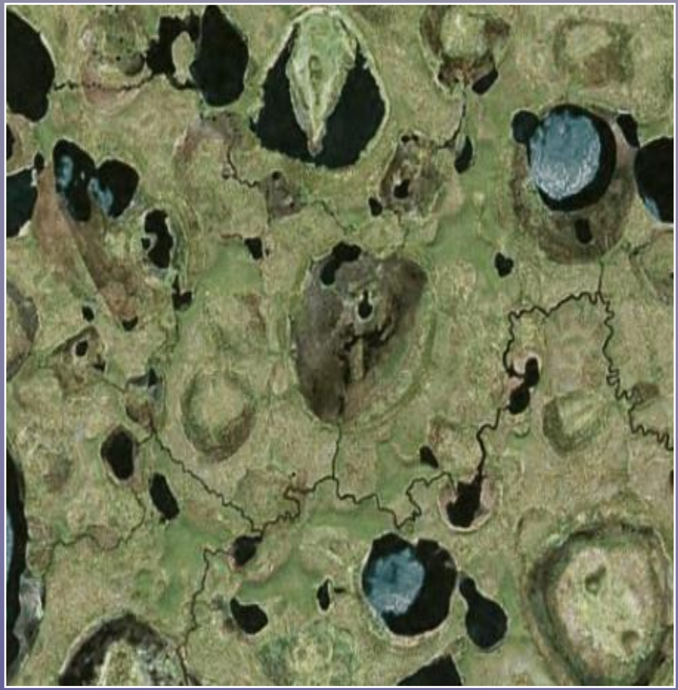
Limit value for average lake area

$$s_l(\infty) = -\frac{1}{\gamma Ei(-\gamma\varepsilon)} e^{-\gamma\varepsilon}$$

where a, λ, γ - parameters, $I = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^2 e^{-\pi\gamma x^2} f_0(x,u)\,dx\,du$

$Ei(x)$ is an integral exponential function, $Ei(-x) = \int_{-\infty}^{x} \frac{e^{-u}}{u} du$

Mathematical model for lacustrine thermokarst plains with fluvial erosion. Analysis of the model



Limit distribution for lake areas (distribution density) - the integral exponential distribution

$$f_{sl}(x,\infty) = -\frac{1}{xEi(-\gamma\varepsilon)} e^{-\gamma x}, \quad x \geq \varepsilon$$

Limit area distribution for khasyreis – exponential distribution

$$F_h(x) = 1 - e^{-\pi\gamma(x-\varepsilon)}$$

where

γ is an average spacing density of erosion sources,

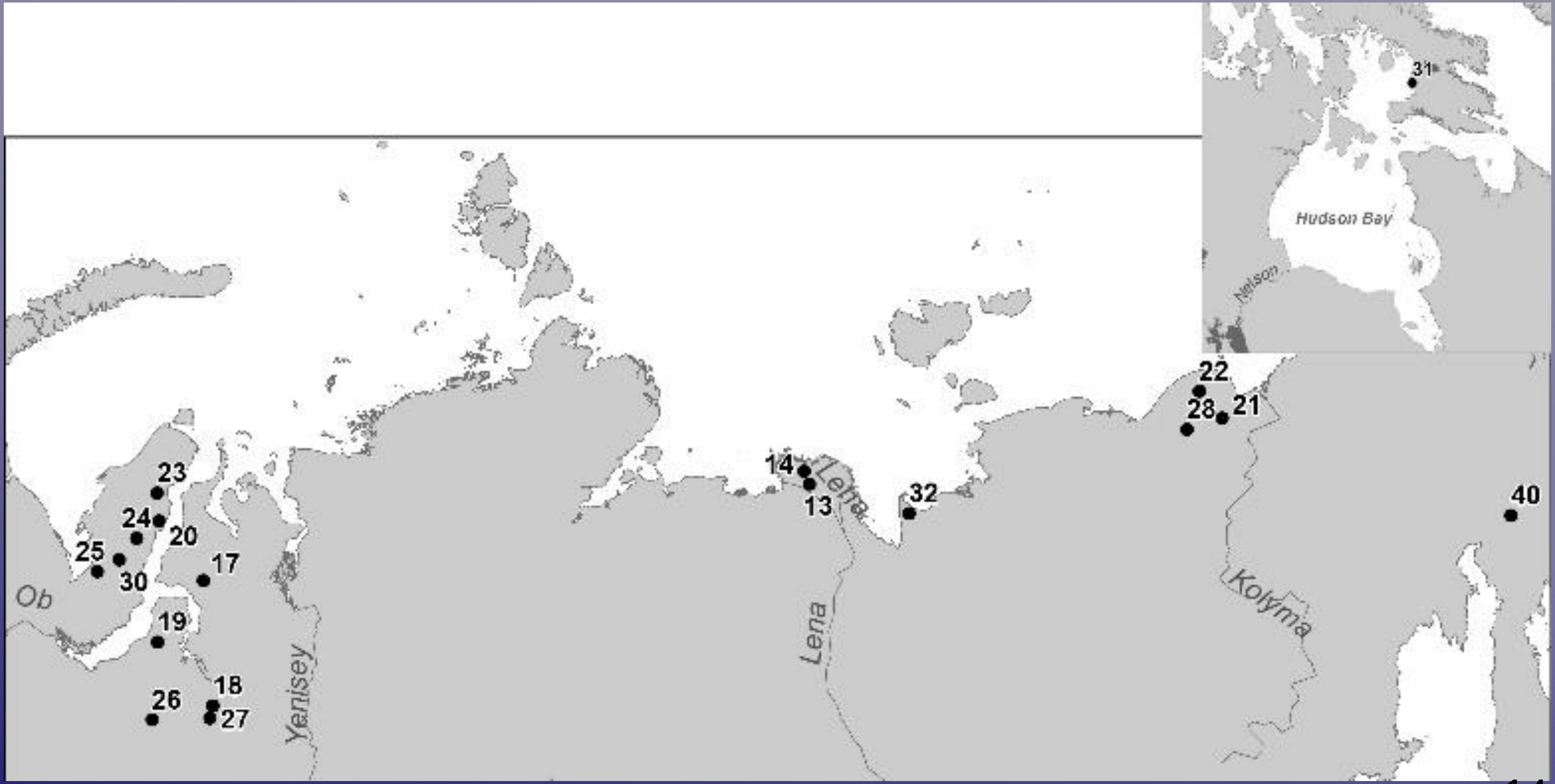
ε is an initial size of a depression,

$Ei(x)$ is the integral exponential distribution

$$Ei(-x) = \int_{-\infty}^{x} \frac{e^{-u}}{u} du$$

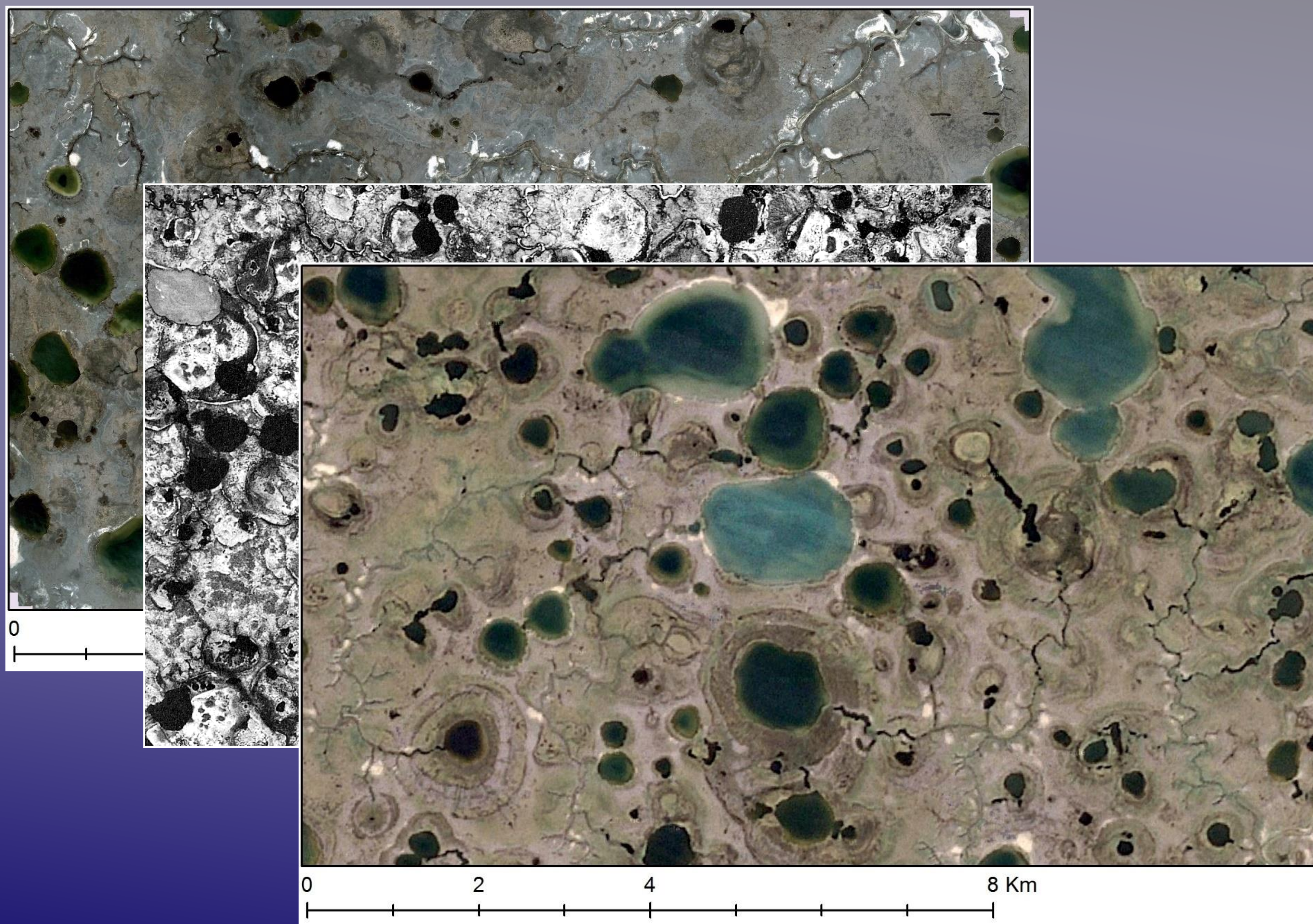
Mathematical for lacustrine thermokarst plains with fluvial erosion. Empirical testing

Key sites located within thermokarst plains with fluvial erosion



Mathematical for lacustrine thermokarst plains with fluvial erosion. Empirical testing

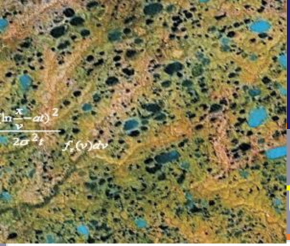
Key sites located in thermokarst plains with fluvial erosion



Mathematical for lacustrine thermokarst plains with fluvial erosion. Empirical testing

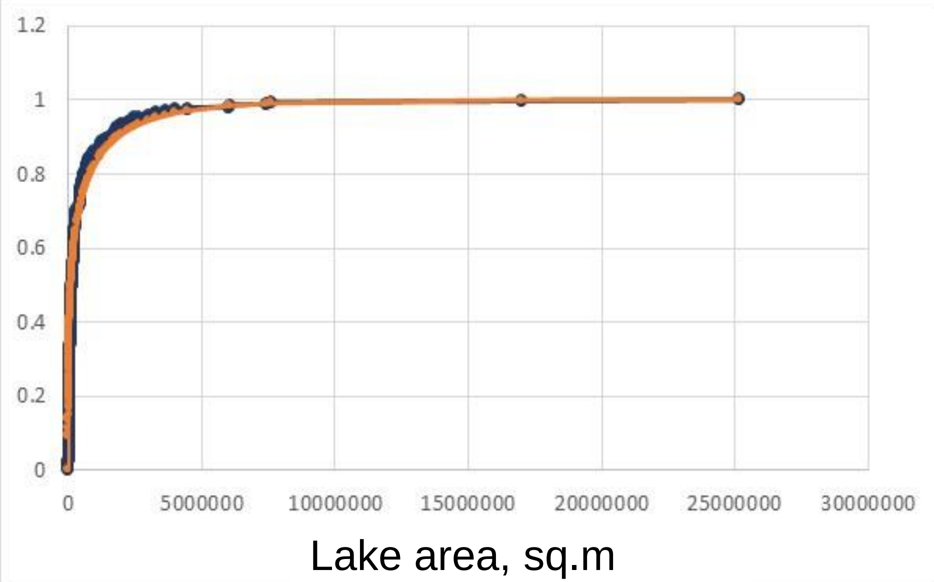
Correspondence between empirical and theoretical distributions of the average thermokarst lake radii

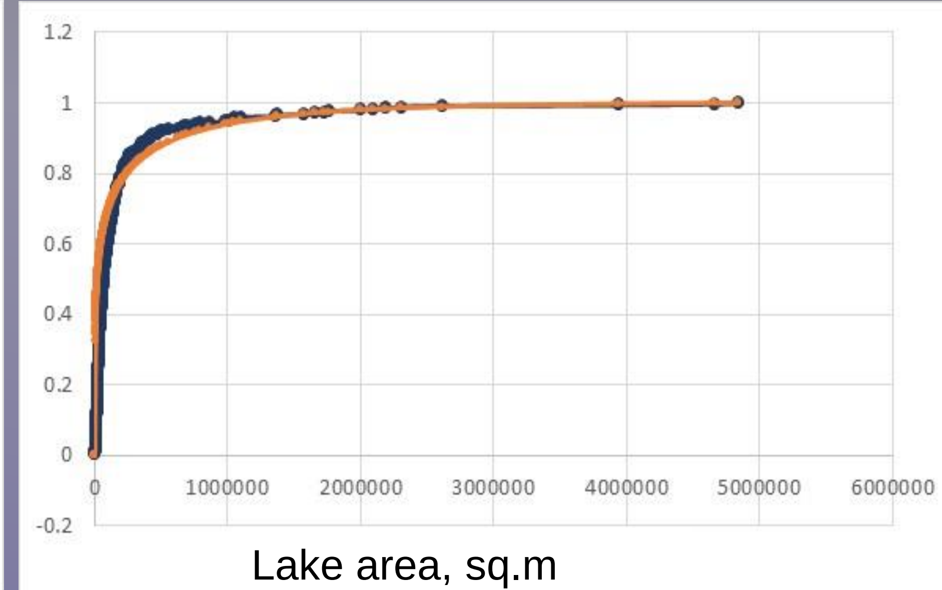
Key site Date sample volume	13-2 581	13-1 598	14-2 209	17- 2 232	18- 2 62	19-2 161	19- 1 160	20- 2 318	20-1 359	21- 2 405
Lognormal	0.000	0.000	**0.014**	0.005	**0.16**	**0.017**	**0.091**	0.007	0.000	0.000
Gamma	0.000	0.000	**0.017**	0.000	**0.018**	0.000	0.000	0.000	0.000	0.000
Normal (for average values)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Integral-exponential	0.000	0.000	**0.022**	0.002	**0.086**	**0.213**	**0.394**	0.000	0.000	**0.109**
Key site Date sample volume	13-2 581	13-1 598	14-2 209	17- 2 232	18- 2 62	19-2 161	19- 1 160	20- 2 318	20-1 359	21- 2 405
Lognormal	**0.010**	0.000	0.000	0.044	0.004	0.001	**0.225**	**0.265**	0.008	**0.310**
Gamma	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Normal (for average values)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Integral-exponential	0.004	**0.014**	**0.641**	**0.220**	**0.663**	**0.024**	0.000	0.000	0.001	**0.053**
Key site Date sample volume	28-1 267	30-2 519	30- 1 519	31- 2 74	31- 1 70	32- 2 430	32- 1 439	40- 2 535		
Lognormal	**0.122**	**0.322**	**0.710**	0.000	0.000	0.000	0.000	0.001		
Gamma	0.000	0.000	0.000	**0.015**	0.000	0.000	0.000	0.000		
Normal (for average values)	0.000	0.000	0.000	0.000	**0.018**	0.000	0.000	0.000		
Integral-exponential	**0.085**	**0.245**	**0.023**	0.005	0.001	0.000	0.000	**0.122**		

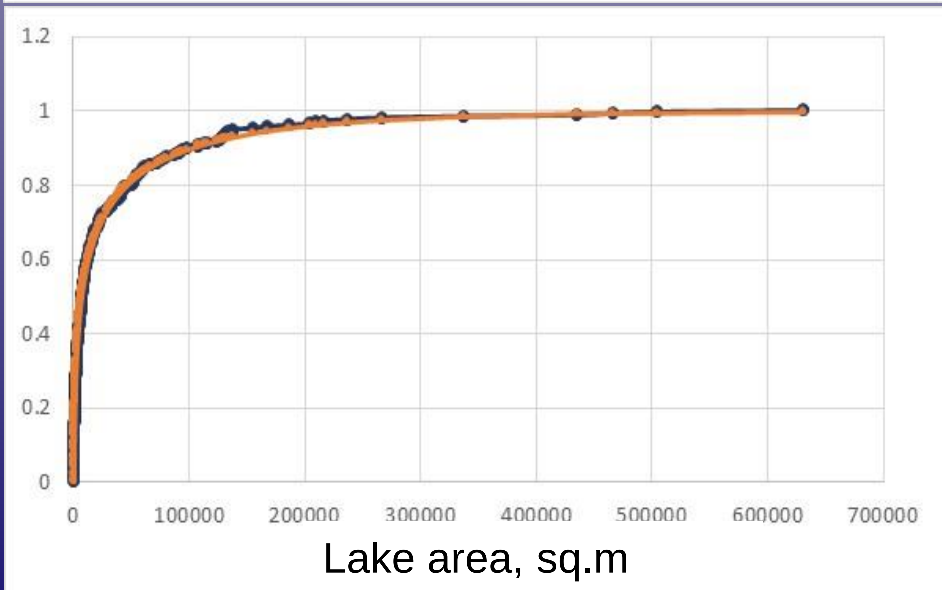


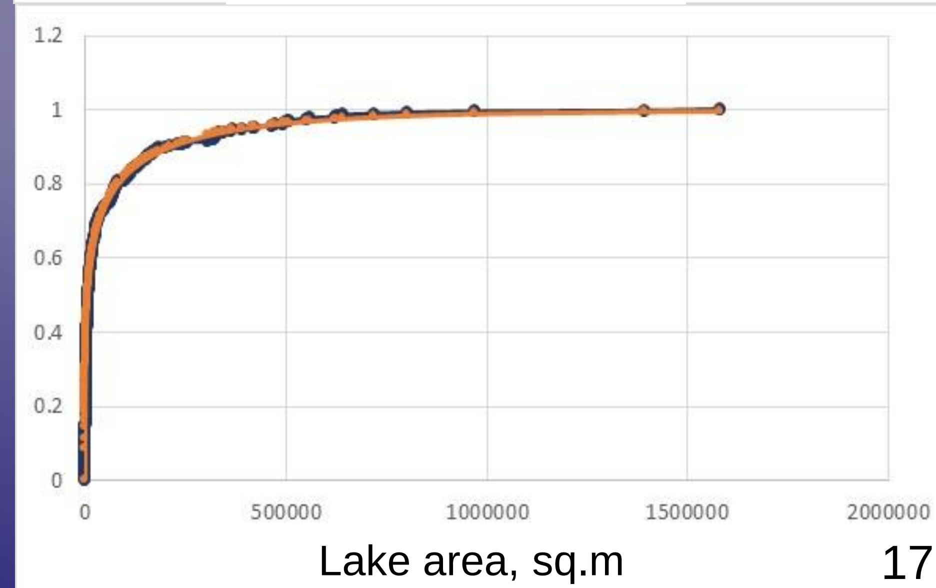
Mathematical for lacustrine thermokarst plains with fluvial erosion. Empirical testing

Correspondence between empirical distributions of the thermokarst lake areas and the integral exponential distribution









Mathematical for lacustrine thermokarst plains with fluvial erosion. Empirical testing

Correspondence between empirical and theoretical distributions of the thermokarst lake areas

Key site Date sample volume	13-2 581	13-1 598	14-2 209	17- 2 232	18- 2 62	19-2 161	19- 1 160	20- 2 318	20-1 359	21- 2 405
Lognormal	0.000	0.000	**0.014**	0.005	**0.16**	**0.017**	**0.091**	0.007	0.000	0.000
Gamma	0.000	0.000	**0.017**	0.000	**0.018**	0.000	0.000	0.000	0.000	0.000
Normal (for average values)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Integral-exponential	0.000	0.000	**0.022**	0.002	**0.086**	**0.213**	**0.394**	0.000	0.000	**0.109**
Key site Date sample volume	**21-1 339**	**22- 2 244**	**22- 1 337**	**23- 2 257**	**24-2 346**	**24- 1 376**	**25- 2 278**	**25-1 281**	**26-2 500**	**28-2 264**
Lognormal	**0.010**	0.000	0.000	0.044	0.004	0.001	**0.225**	**0.265**	0.008	**0.310**
Gamma	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Normal (for average radii)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Integral-exponential	0.004	**0.014**	**0.641**	**0.220**	**0.663**	**0.024**	0.000	0.000	0.001	**0.053**
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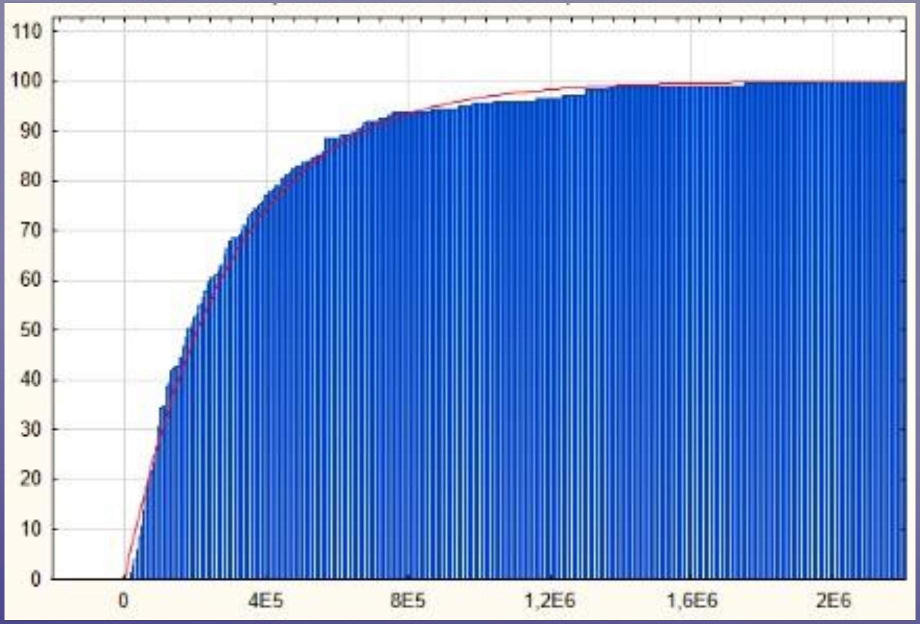
Mathematical for lacustrine thermokarst plains with fluvial erosion. Empirical testing

$$F_h(x) = 1 - e^{-\gamma x}$$



a

b

Graphs demonstrating the correspondence of empirical distributions of the khasyrei areas to the exponential distribution for key sites A3 (a) and A8 (b)

Conclusions

The model fitting the asynchronous start and the lake growth rate proportional to heat losses through the side surface is relevant for most of the homogenous sections of thermokarst plains with fluvial erosion in different natural environments.

The mathematical model of the morphological pattern for thermokarst plains with fluvial erosion makes it possible to substantiate theoretically and empirically confirm the important relationships between various morphometric parameters in different physiographical conditions.

We have shown that the behavior of the thermokarst lake areas within the thermokarst plains with fluvial erosion obeys the integral exponential distribution and that of the khasyrei areas - exponential.

The morphological pattern of the thermokarst plains with fluvial erosion can be in the state of the dynamic balance; natural risk assessment and prognosis studies should take it into account.

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Mathematical morphology of clryolithozone landscape
Moscow
2016