THE AUSTRALIAN NATIONAL UNIVERSITY

Reflections on adding the Z dimension to earth system analysis

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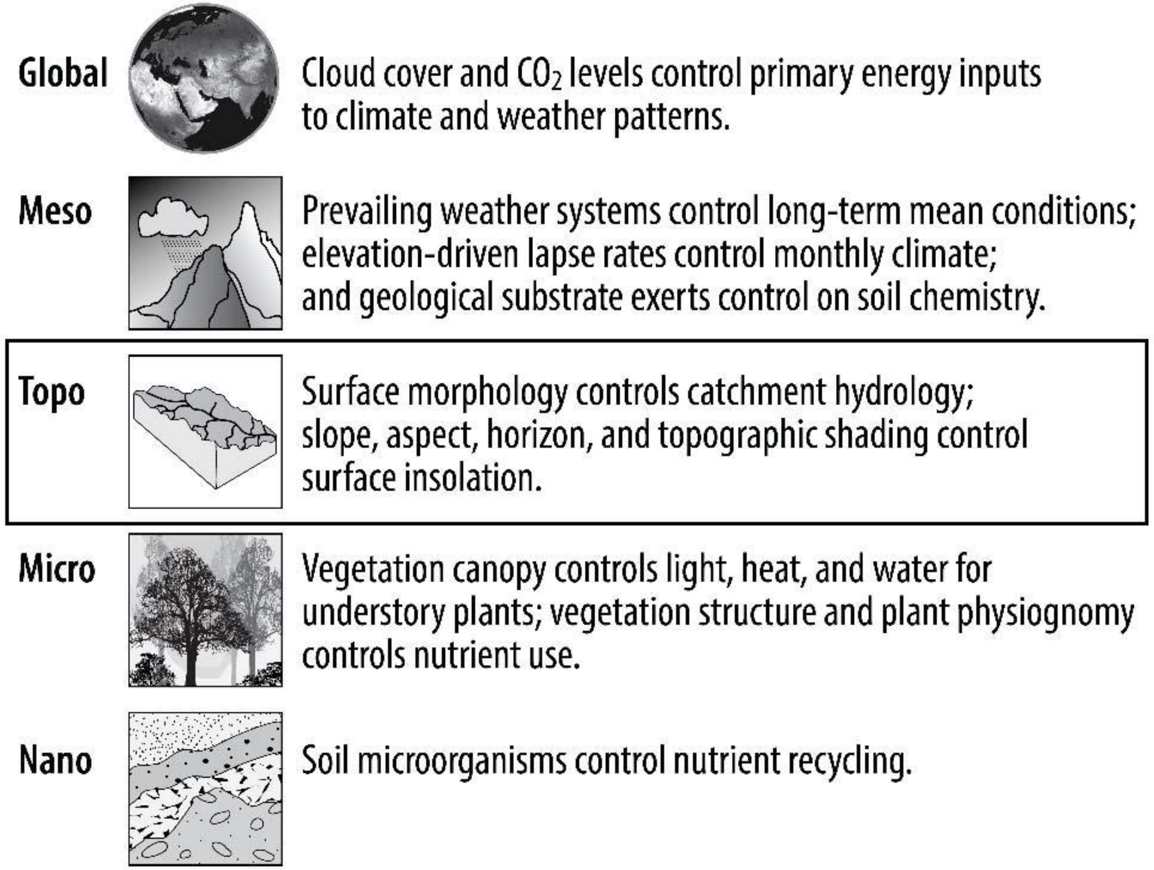
Geomorphometry 2021

Australian National University

Contents

- Mesoscale and Toposcale Context
- Surface Climate – topographic dependence
- Evolution of interpolation of DEMs by ANUDEM – surface shape and drainage structure
- Locally adaptive multi-grid method with automatic drainage enforcement, data sources – streams, lakes, contours, cliffs
- Recent applications to dense noisy source elevation data
- Specific catchment area – theory and calculation
- Conclusion

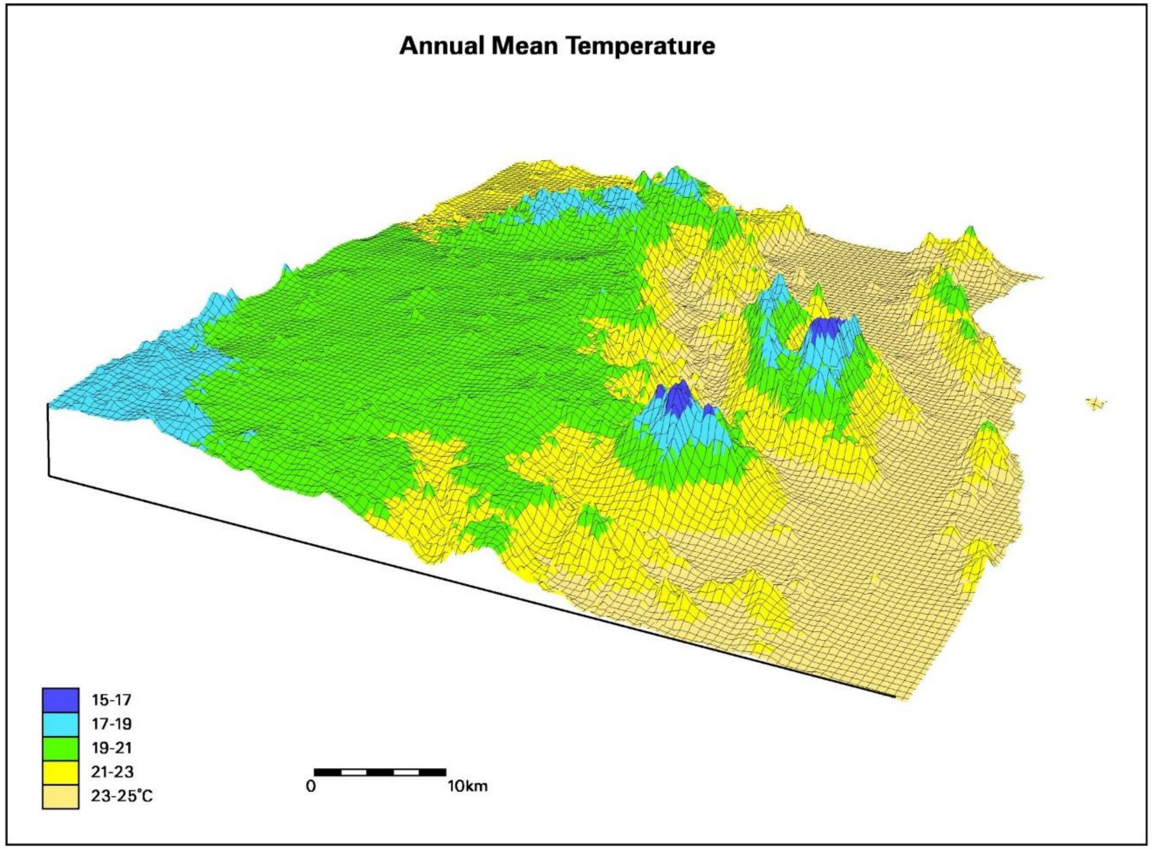
Scales of biophysical processes



Global	Cloud cover and CO_2 levels control primary energy inputs to climate and weather patterns.
Meso	Prevailing weather systems control long-term mean conditions; elevation-driven lapse rates control monthly climate; and geological substrate exerts control on soil chemistry.
Topo	Surface morphology controls catchment hydrology; slope, aspect, horizon, and topographic shading control surface insolation.
Micro	Vegetation canopy controls light, heat, and water for understory plants; vegetation structure and plant physiognomy controls nutrient use.
Nano	Soil microorganisms control nutrient recycling.

Wilson 2016 *Environmental Applications of Digital Terrain Modeling*

Mesoscale and Toposcale Representations of Surface Processes



Trivariate smoothing spline model for interpolating elevation dependent climate data

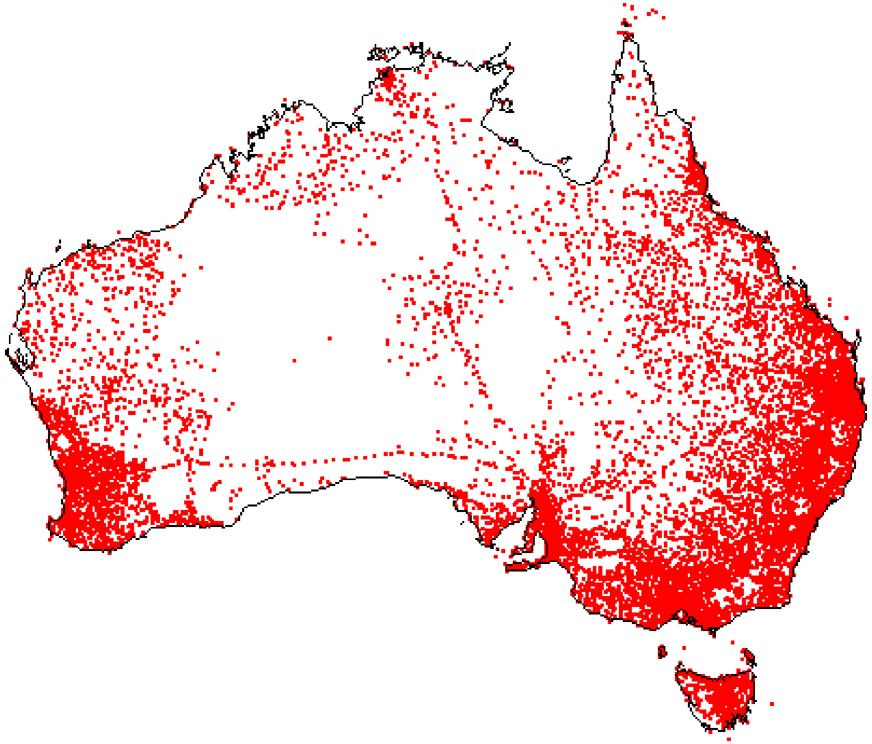
Model: $z = f(x,y,h) + \varepsilon$

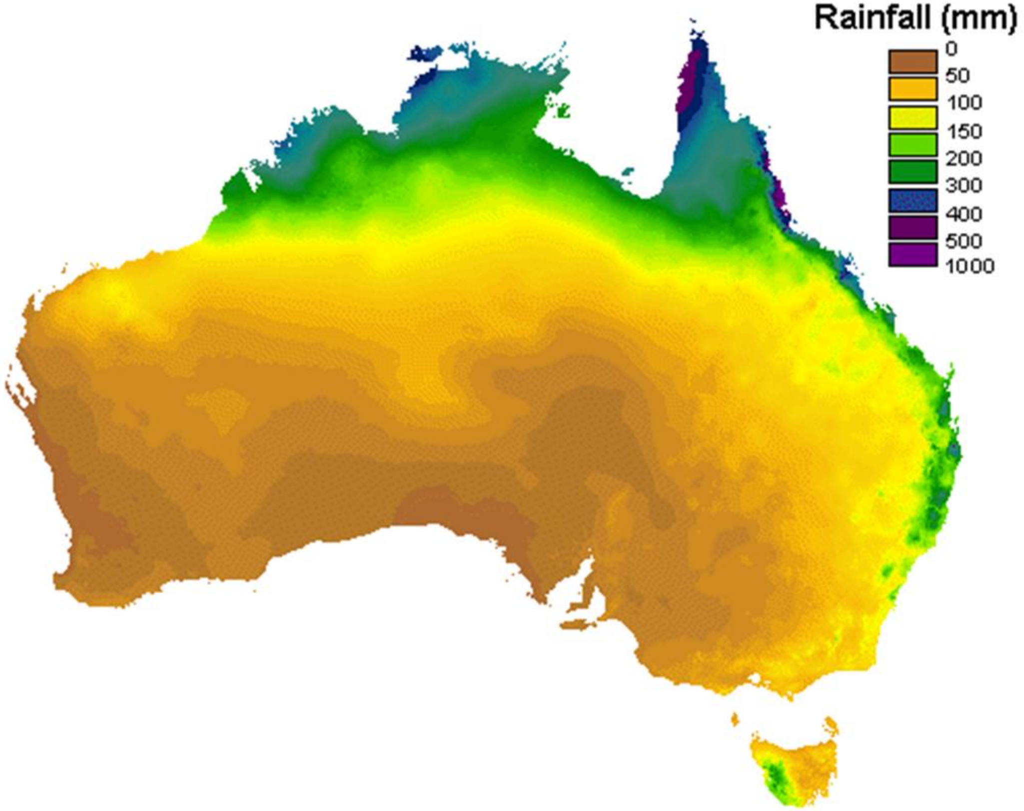
Solution:

Minimise $\|z - f\|^2 + \lambda J_2(f)$

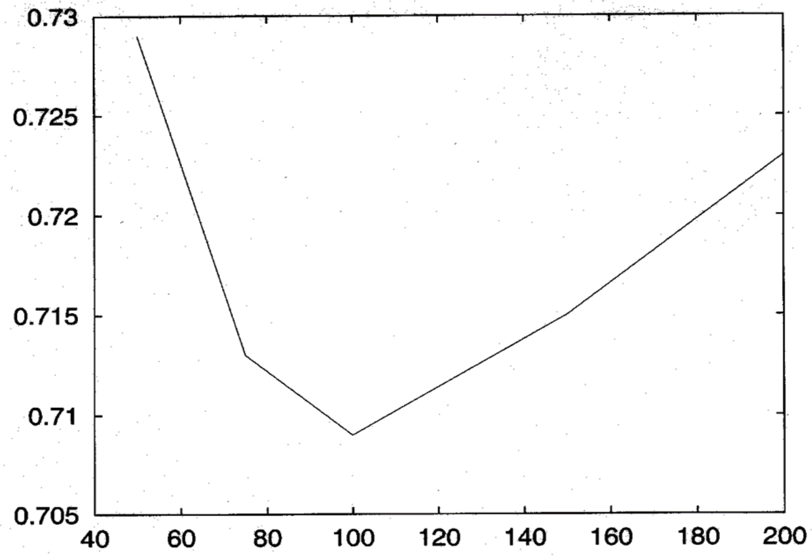
Wahba and Wendelberger 1980 *Journal of Applied Meteorology*
Hutchinson and Bischof 1983 *Australian Meteorological Magazine*

January mean rainfall across Australia from 12,000 points

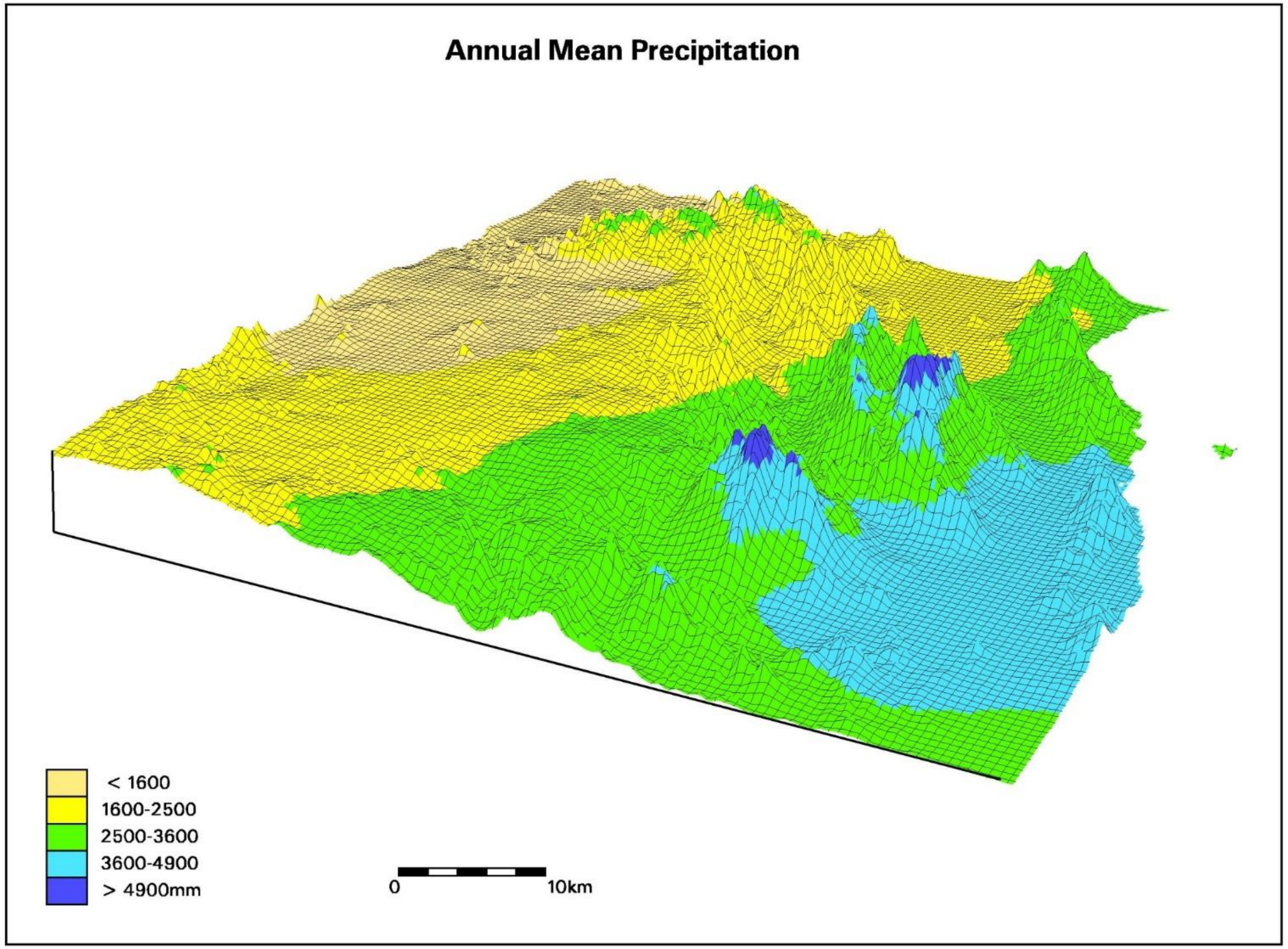


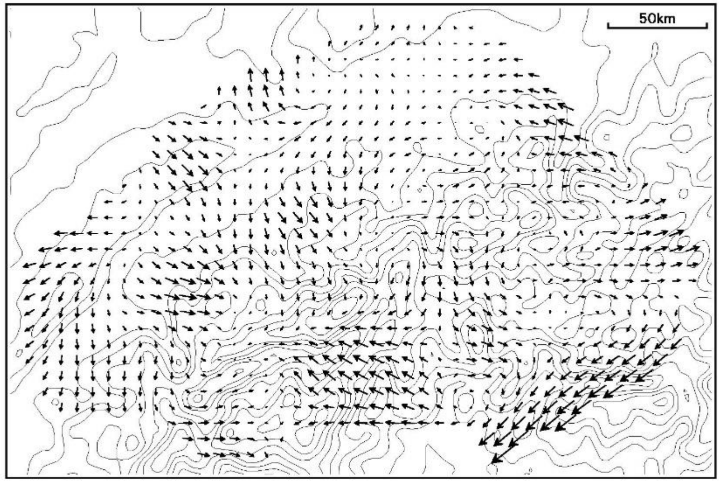


Topographic Scale and Aspect Relationships for Optimum Spatial Representation of Rainfall



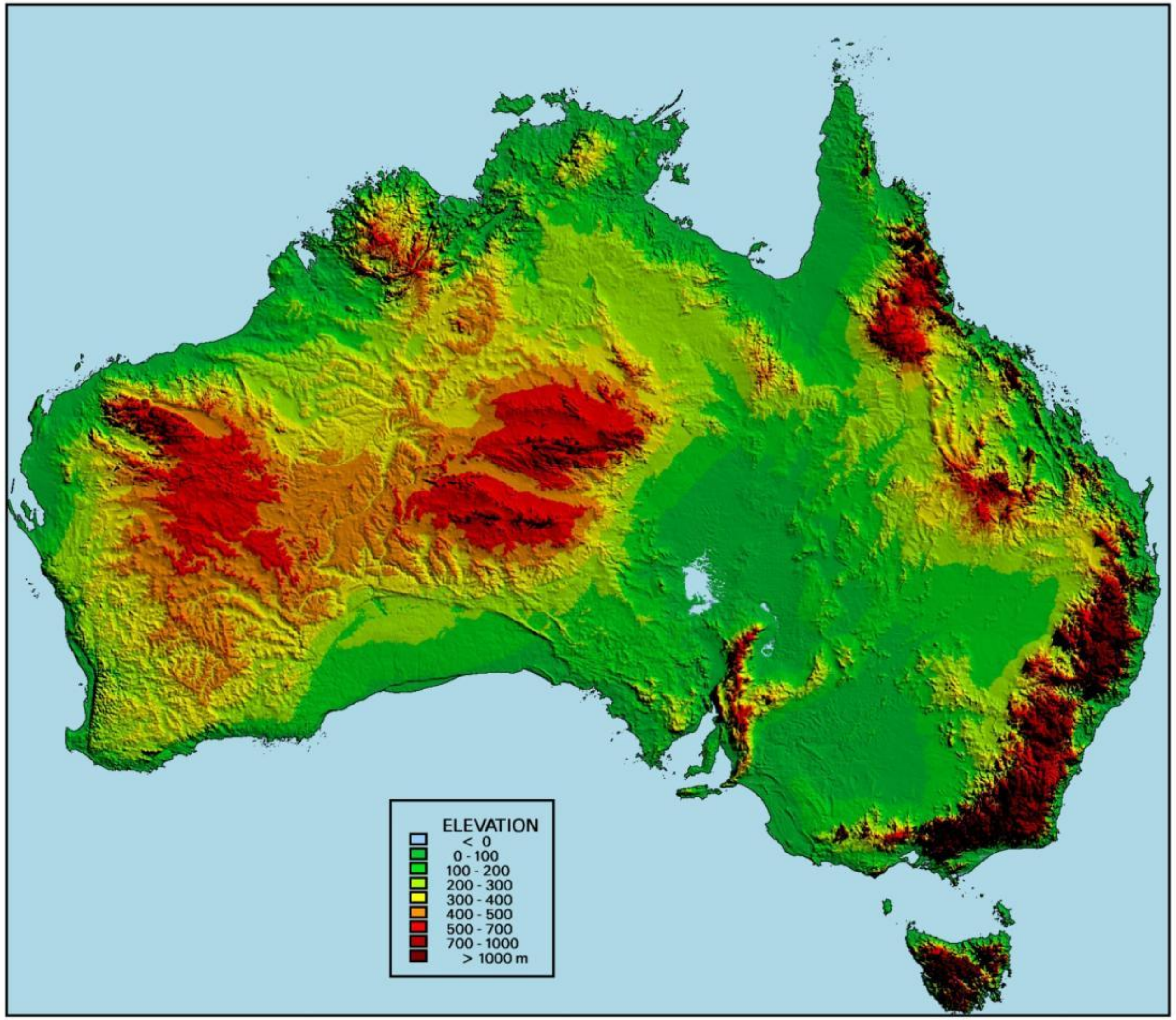
RMS validation error ($mm^{1/2}$) as a function of scaling of 10 km resolution DEM elevations.





Topographic aspect dependence

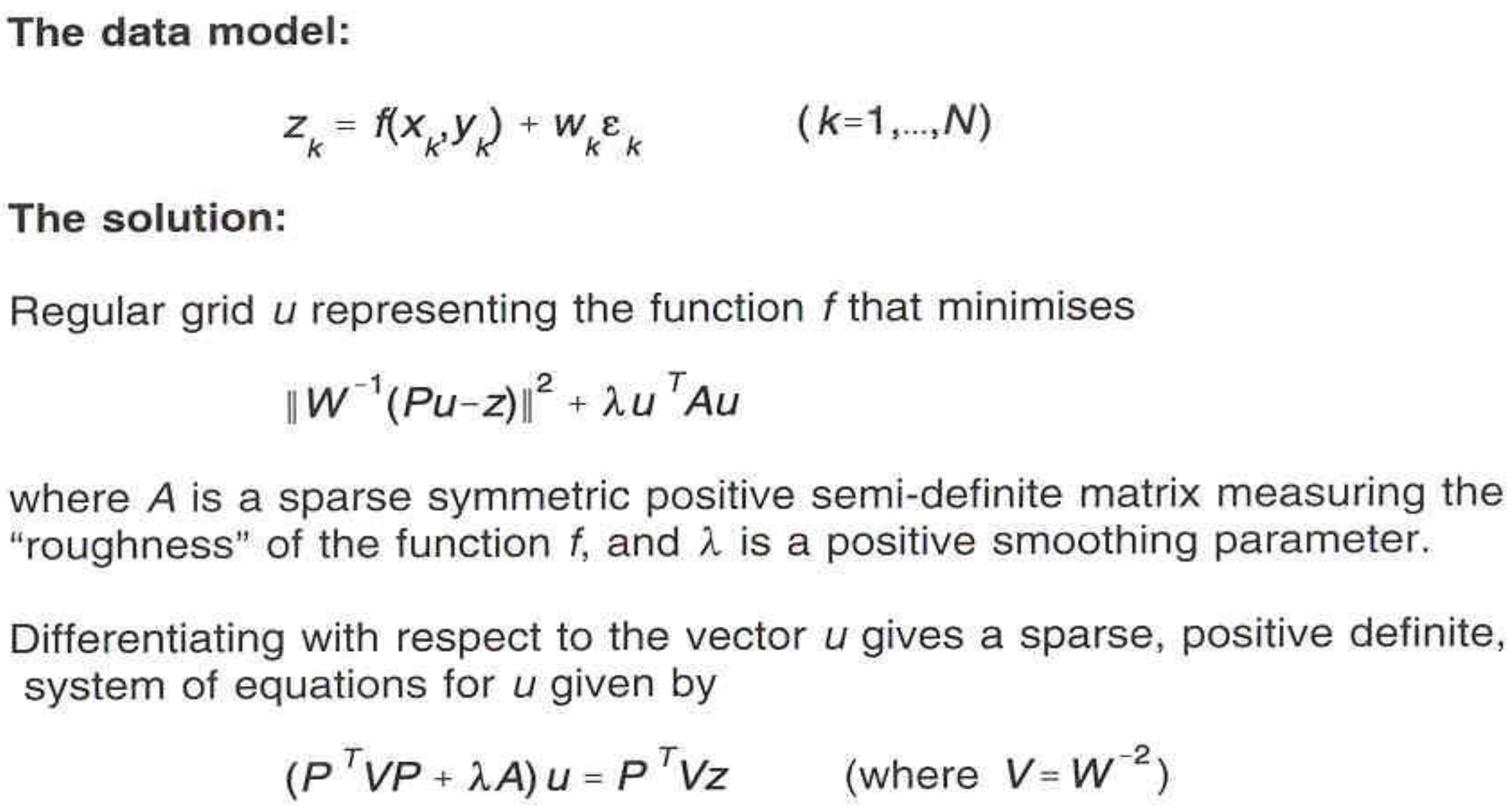
9 Second (250m) Australian DEM Version 3   2008



1976-82	First coarse scale Aust DEM - BMR
1965-88	Digitising 1:100K maps - AUSLIG
1983-88	Early development of ANUDEM
1991	First drainage enforced Aust DEM
1996	9 second DEM Version 1
1998	National Wild Rivers Study - CRES
1988-00	Further development of ANUDEM
2001	9 second DEM Version 2 – with AUSLIG
2001-05	Further development of ANUDEM
2005	9 second DEM Version 3 – with Geoscience Australia

Hutchinson, Trevor Dowling (1991), John Stein and Janet Stein (1996-2008)

Underlying iterative multigrid finite difference interpolation algorithm



The data model:

$$z_k = f(x_k, y_k) + w_k \varepsilon_k \qquad (k=1,\ldots,N)$$

The solution:

Regular grid u representing the function f that minimises

$$\| W^{-1}(Pu - z) \|^2 + \lambda u^T A u$$

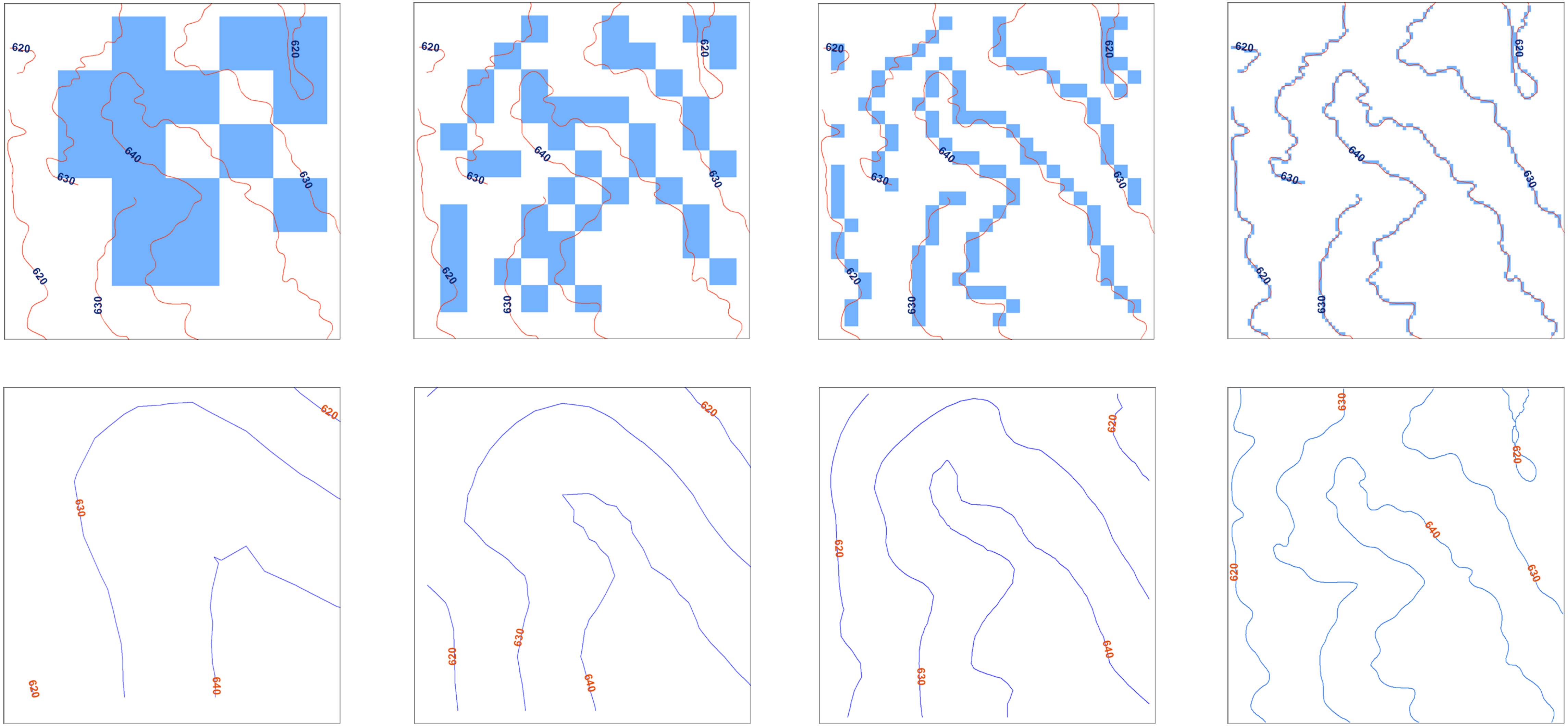
where A is a sparse symmetric positive semi-definite matrix measuring the "roughness" of the function f, and λ is a positive smoothing parameter.

Differentiating with respect to the vector u gives a sparse, positive definite, system of equations for u given by

$$(P^T V P + \lambda A)\, u = P^T V z \qquad (\text{where } V = W^{-2})$$

Briggs 1974 *Geophysics*
Torgersen, Hutchinson et al 1983 *Paleogeography, Paleoclimatology, Paleoecology*
Hutchinson 1989 *Journal of Hydrology*

Nested (Multigrid) Interpolation

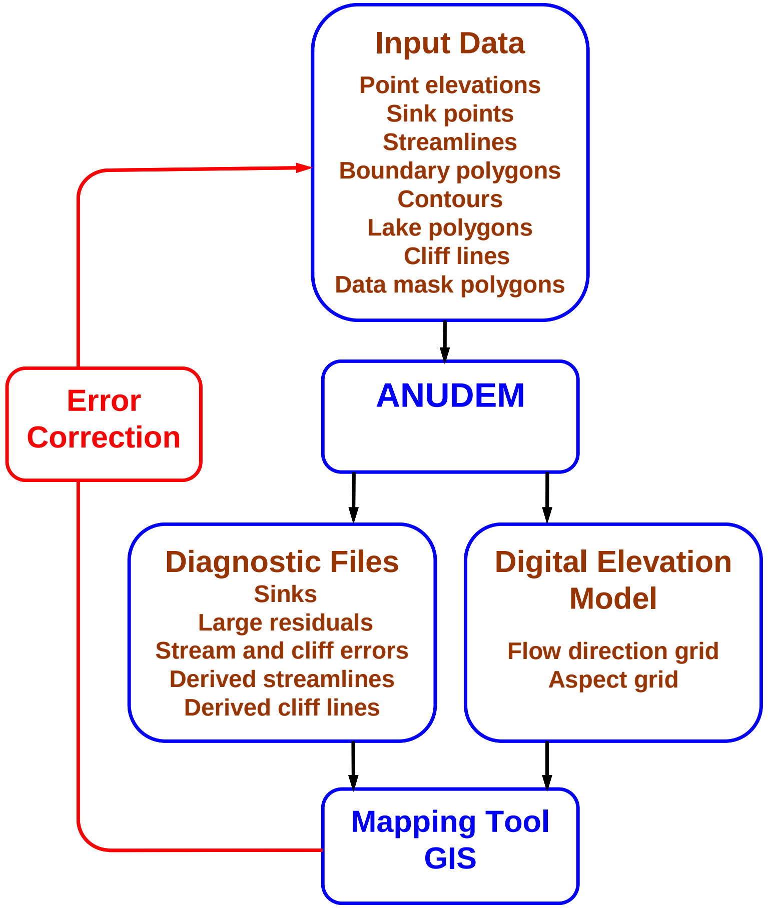


Locally Adaptive Modifications to the Finite Difference Interpolation Method

- Data values weighted according to their discretization error – defined by local slope
- Cell to cell constraints are applied to respect natural conditions implied by stream lines, lake boundaries, ridge lines and cliffs
- Roughness penalty is locally modified to respect these cell to cell constraints
- Yields stable convergence with minimal reliance on hard constraints

Data Flows for the ANUDEM Elevation Gridding Program

Version 5.3 = Topo to Raster in ArcGIS



Input Data
Point elevations
Sink points
Streamlines
Boundary polygons
Contours
Lake polygons
Cliff lines
Data mask polygons

ANUDEM

Error Correction

Diagnostic Files
Sinks
Large residuals
Stream and cliff errors
Derived streamlines
Derived cliff lines

Digital Elevation Model
Flow direction grid
Aspect grid

Mapping Tool GIS

Progressively upgraded from 1983 to around 2011

Automatic drainage enforcement

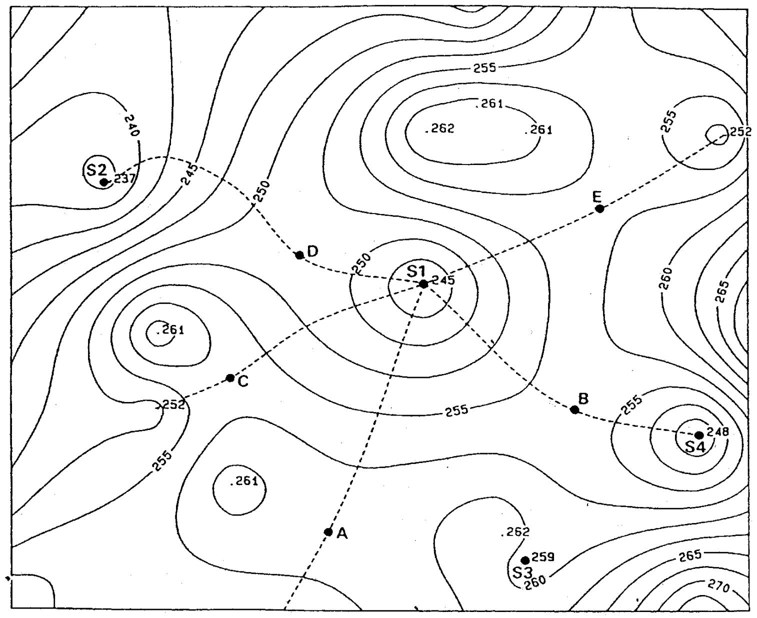


Fig. 4. Example showing how the saddle points *A*, *B*, *C*, *D*, *E* are associated with the sink point *S*
via flow lines which are indicated by dashed lines. Additional sink points are denoted by *S*2, *S*3
*S*4. Data points are indicated by their height in metres.

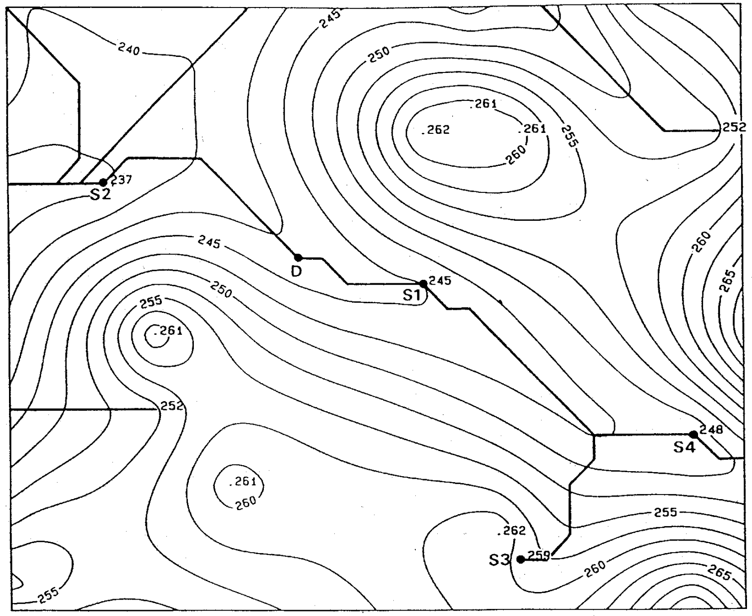
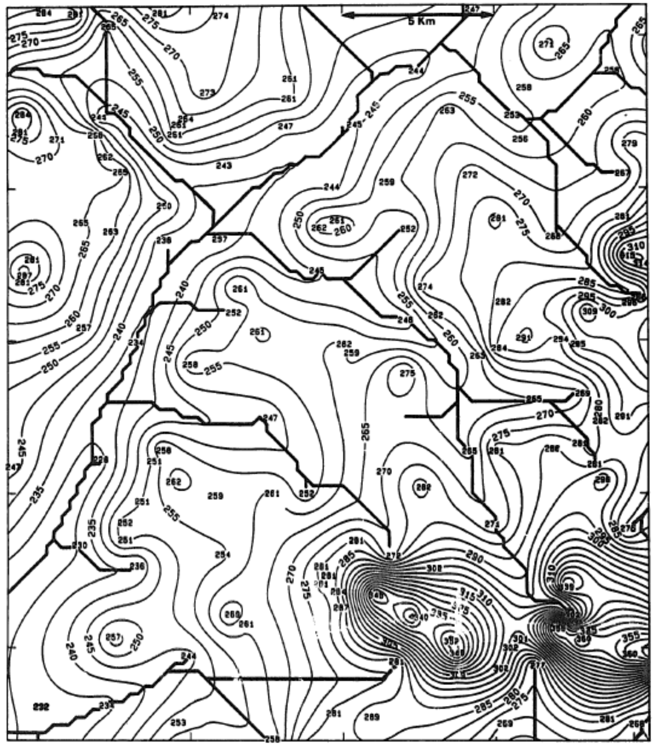


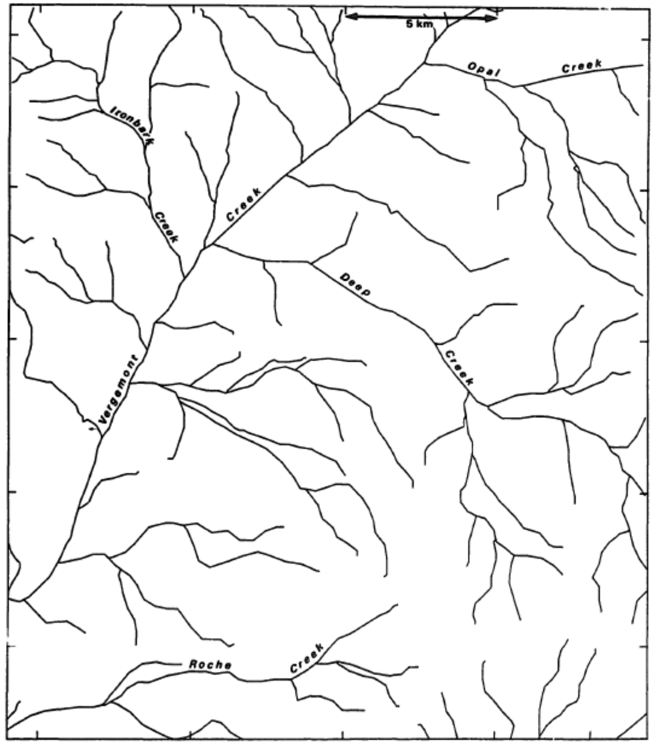
Fig. 5. The result of drainage enforcement applied to the example of Fig. 4. Piecewise linear lines indicate inferred drainage lines.

Hutchinson 1989 *Journal of Hydrology*

Validation of automatic drainage enforcement

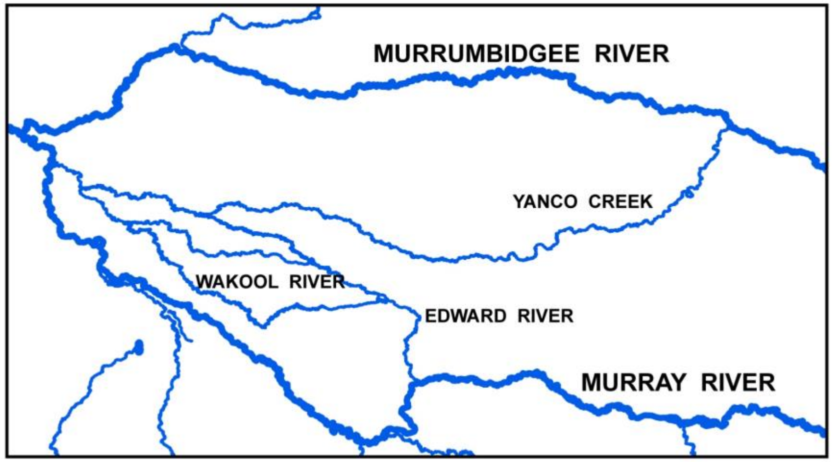


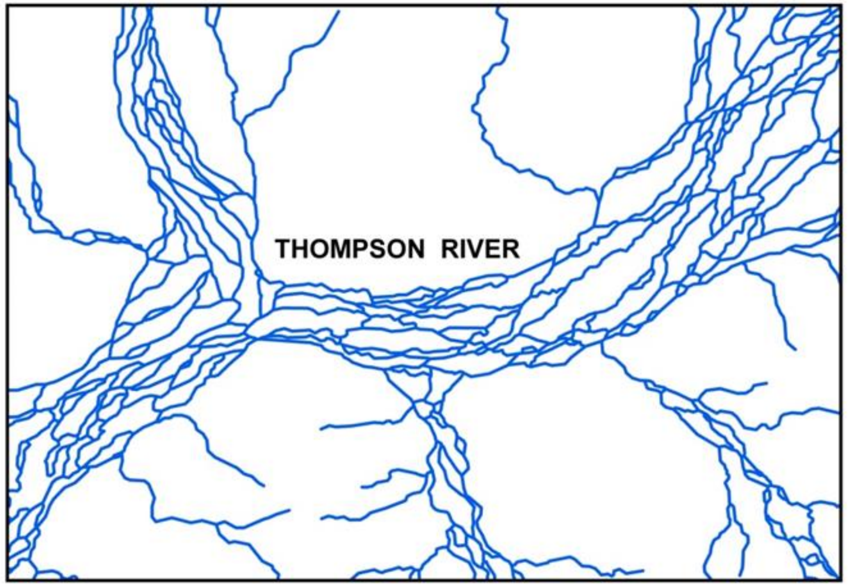
Approx 120 elevation data points - only

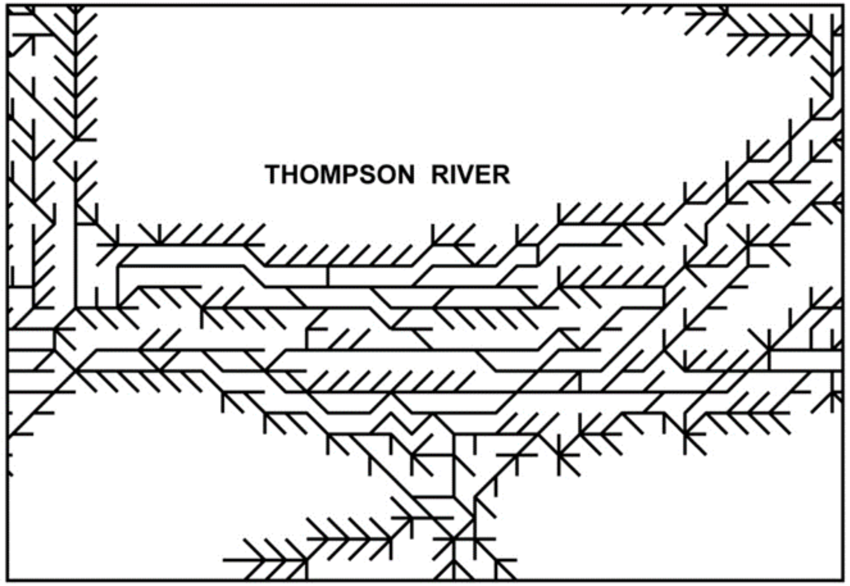


Principal streams well matched by automated drainage enforcement

Incorporation of Anabranching Systems and Braided Streams

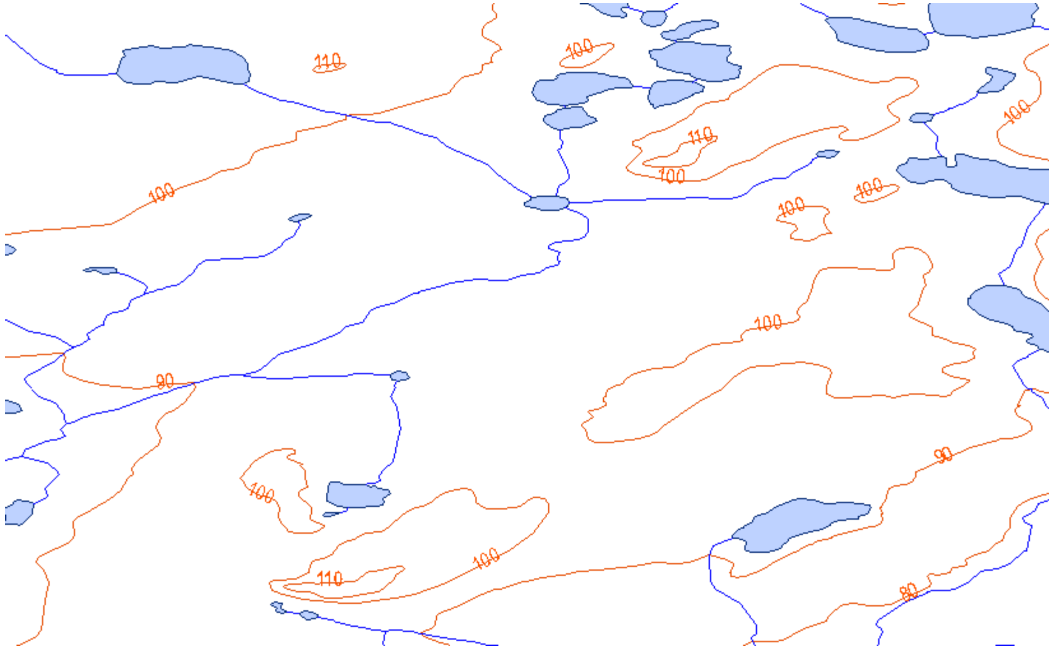


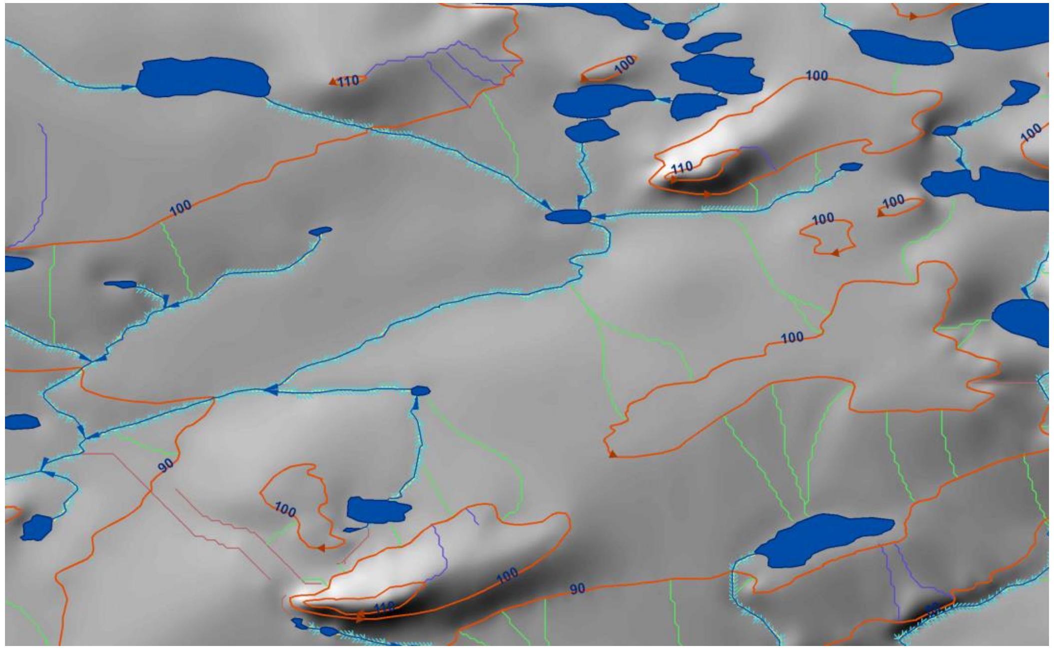




Hutchinson, Janet Stein 2001 - Australian 9 second DEM Version 2

Northern Canada – gridding sparse contour, stream and lake data

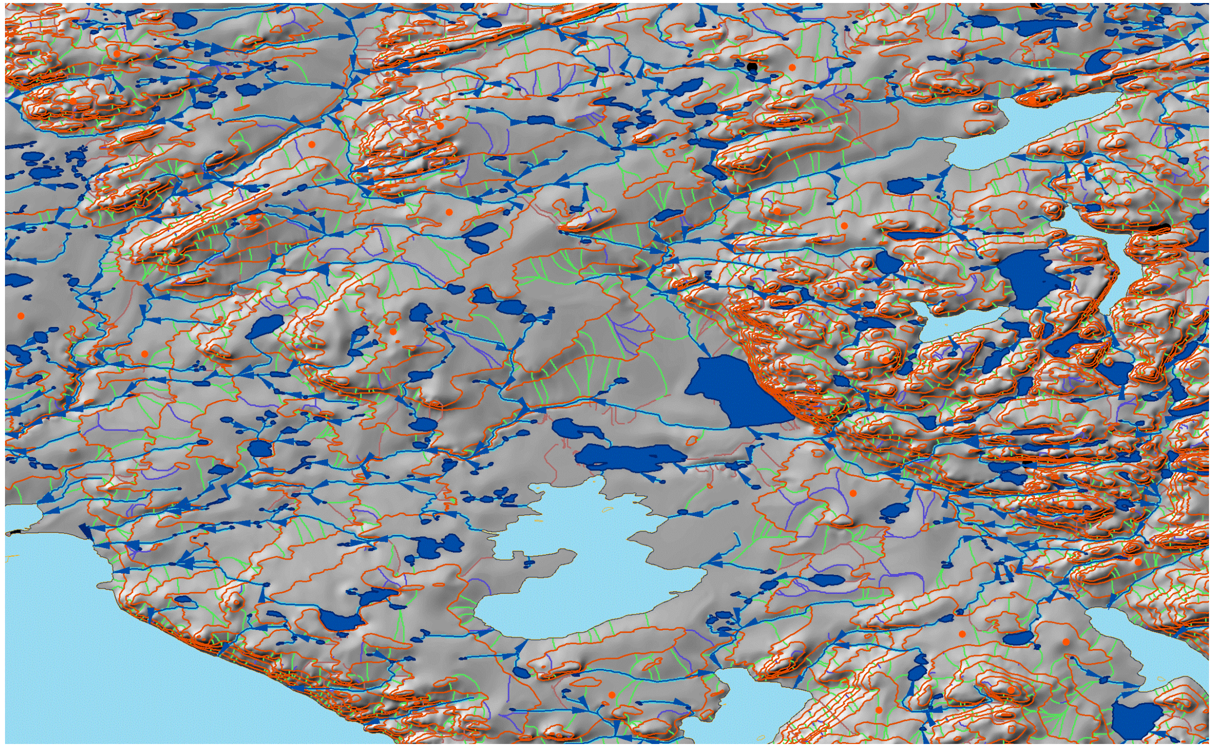


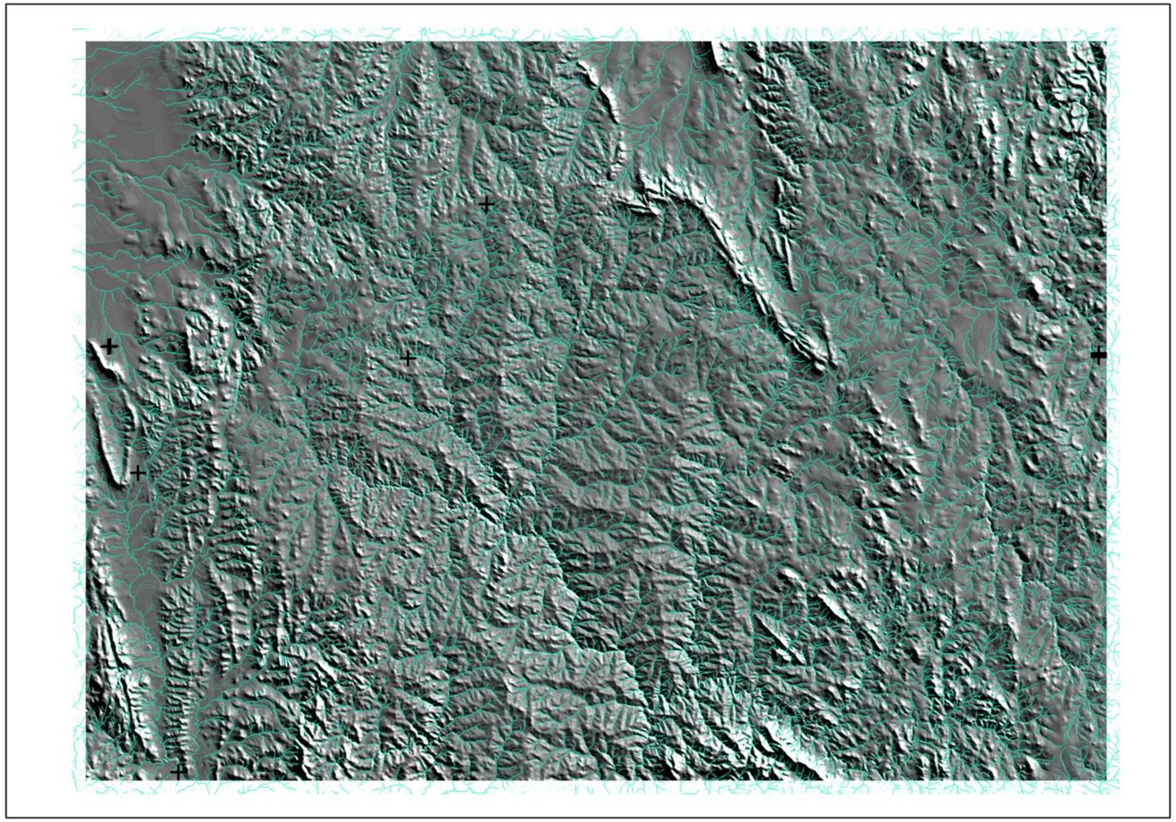


Hutchinson 1988 *Spatial Data Handling Symposium* - Contours
Hutchinson and Xu 2006 *DEM for Canada* – Lakes with interconnecting Streams

Northern Canada region - essentially sink free

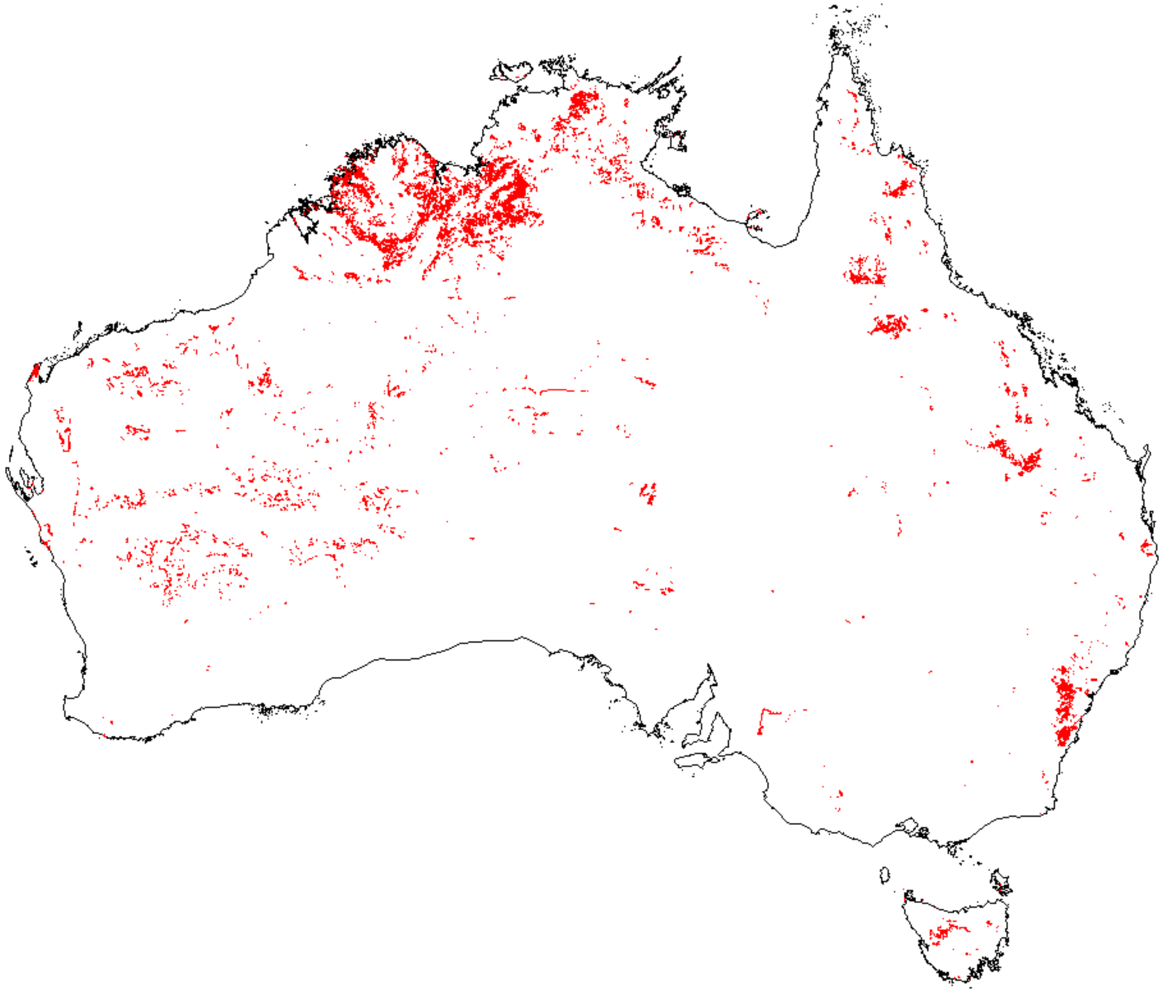
ACT Australia region – from 1:25K contours and streams - 5 sinks



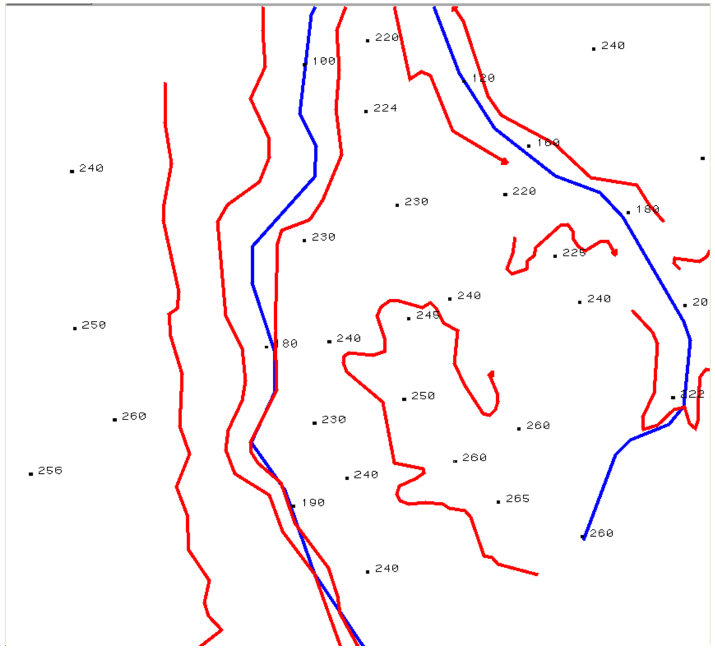


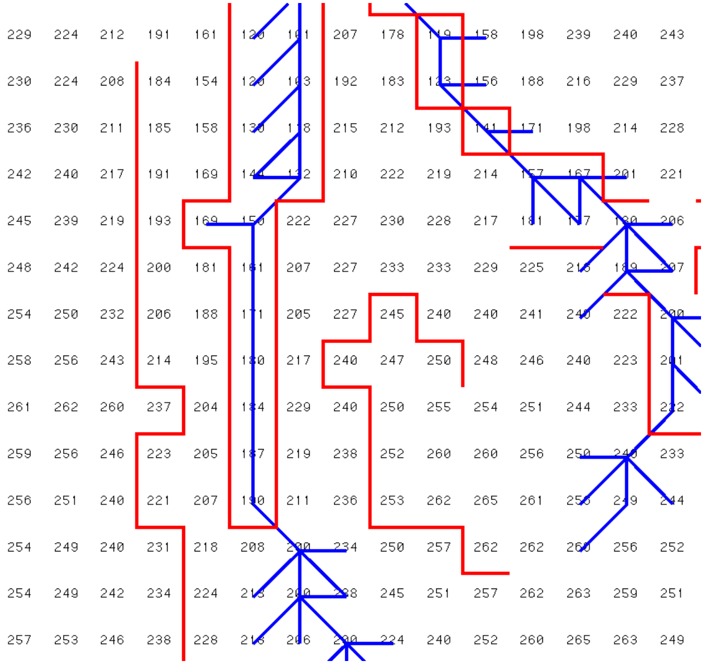
ANUDEM Version 5.3 2006

TOPO250K GEODATA Cliff Lines



Cliff (red) and streamline (blue) data

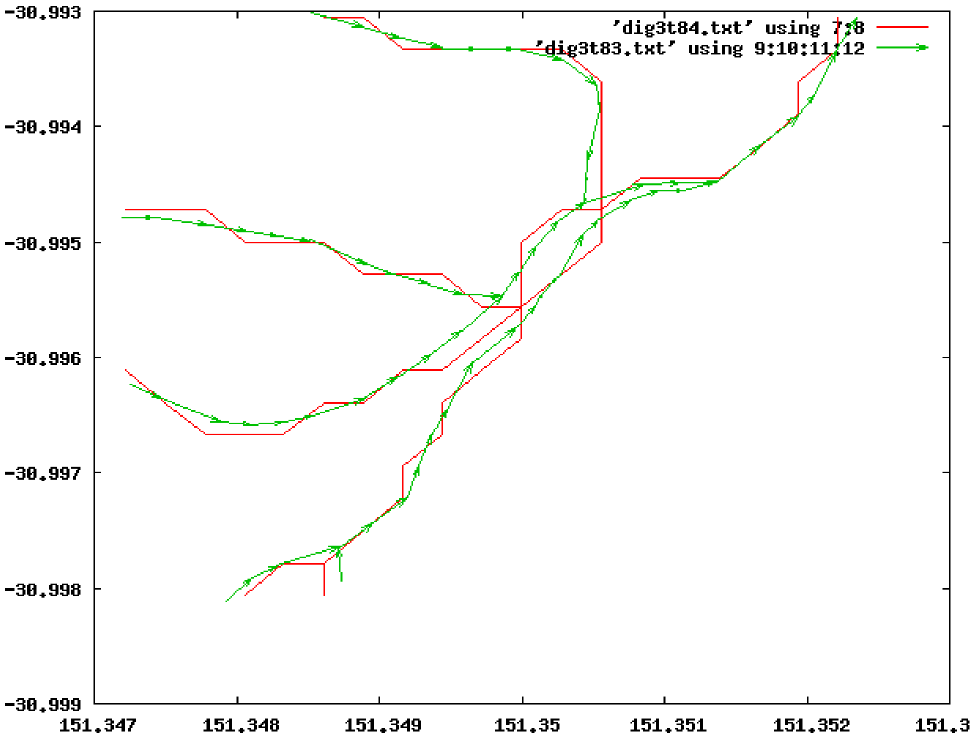


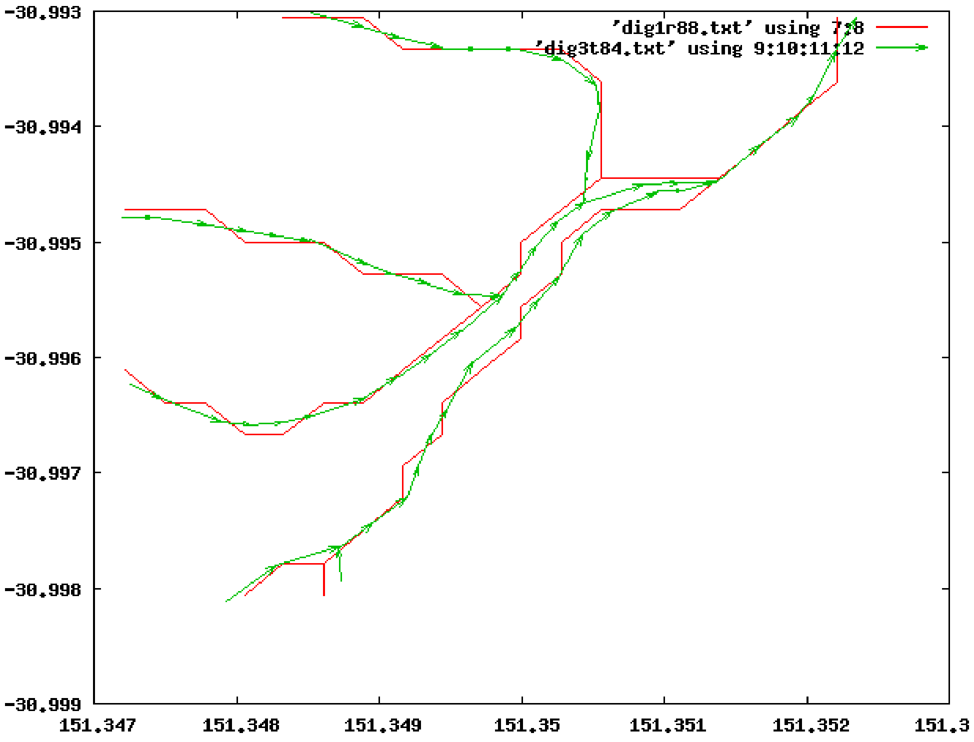


229	224	212	191	161	120	101	207	178	119	158	198	239	240	243
230	224	208	184	154	120	103	192	183	123	156	188	216	229	237
236	230	211	185	158	130	118	215	212	193	141	171	198	214	228
242	240	217	191	169	144	132	210	222	219	214	157	167	201	221
245	239	219	193	169	150	222	227	230	228	217	181	177	190	206
248	242	224	200	181	161	207	227	233	233	229	225	216	189	207
254	250	232	206	188	171	205	227	245	240	240	241	240	222	200
258	256	243	214	195	180	217	240	247	250	248	246	240	223	201
261	262	260	237	204	184	229	240	250	255	254	251	244	233	222
259	256	246	223	205	187	219	238	252	260	260	256	250	240	233
256	251	240	221	207	190	211	236	253	262	265	261	256	249	244
254	249	240	231	218	208	200	234	250	257	262	262	260	256	252
254	249	242	234	224	213	200	238	245	251	257	262	263	259	251
257	253	246	238	228	218	206	200	224	240	252	260	265	263	249

ANUDEM Version 5.3 2006

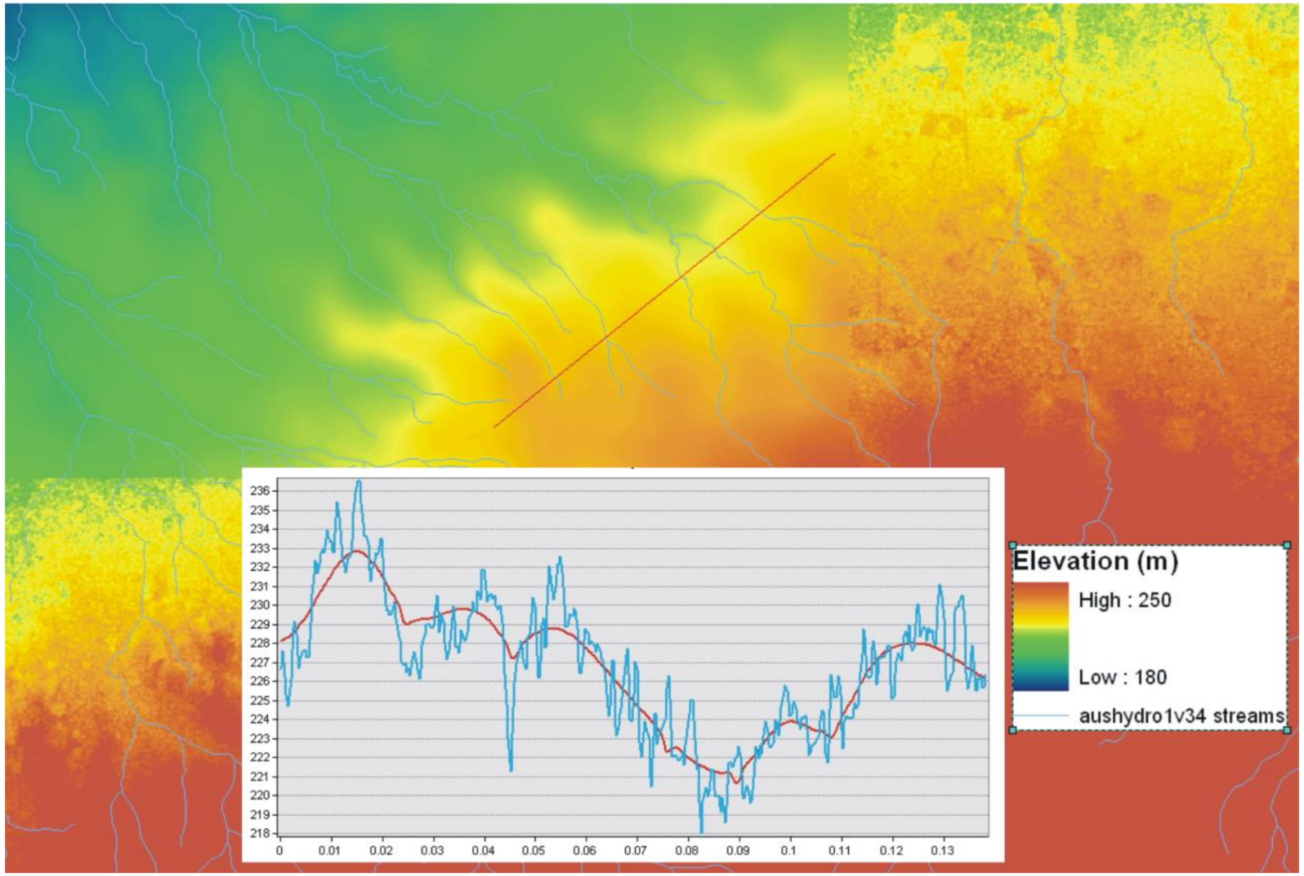
Adjustment of streams and disjunctions to restore correct junction order





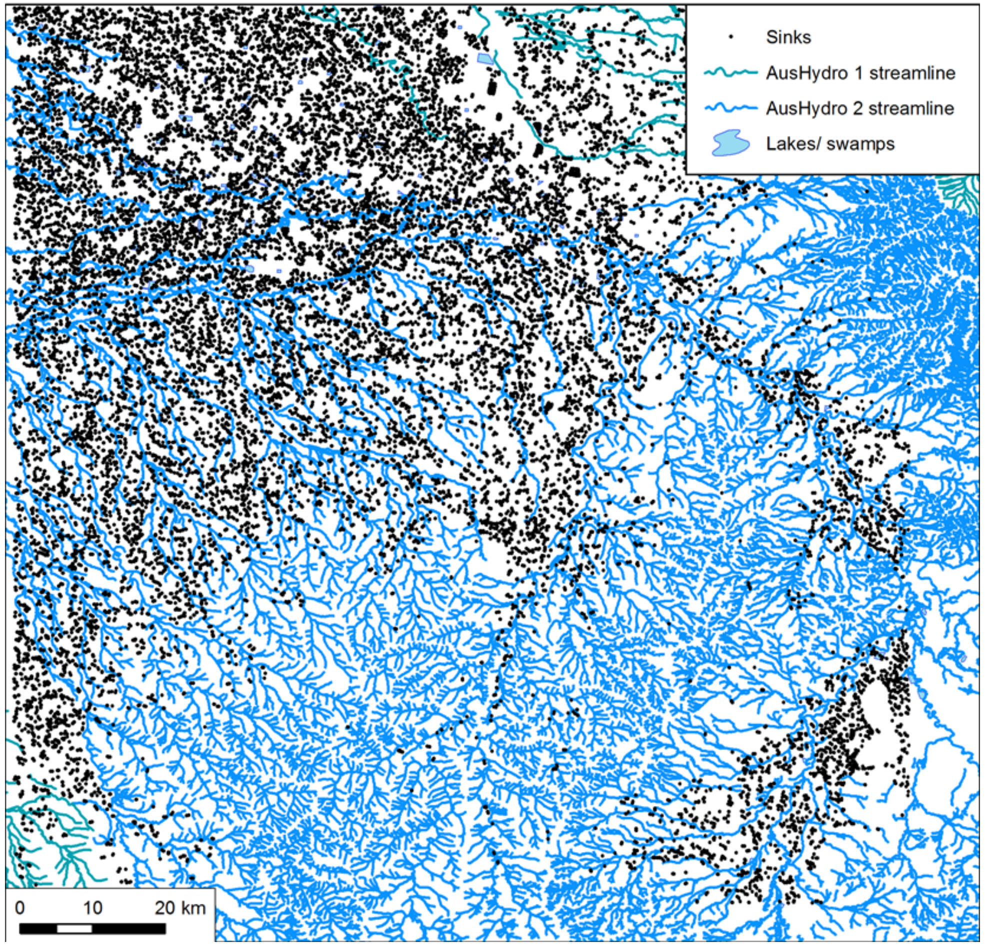
ANUDEM Version 5.4 2011

Smoothing of SRTM Data - 2m Standard Error



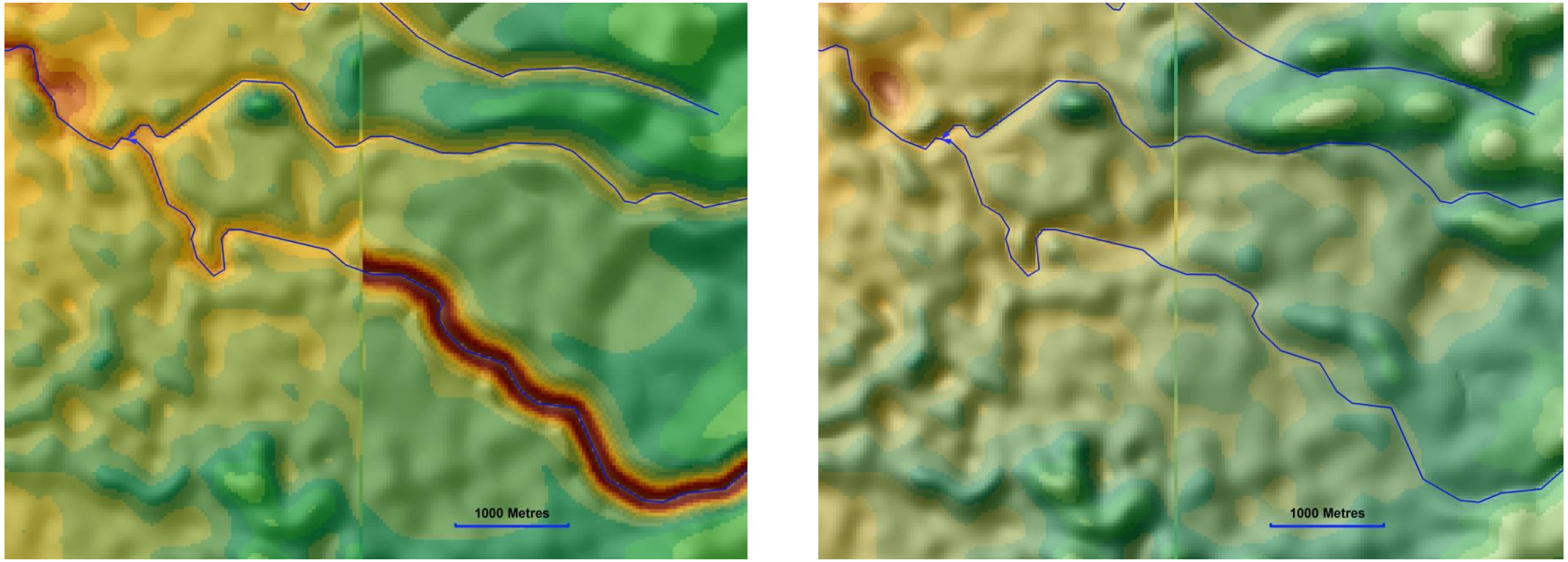
Stream Data and Remaining sinks from SRTM data

DEM-H sinks: Tile E149s31 (Namoi River catchment)



ANUDEM Version 5.4 2011
Gallant et al 2011 Australian 1 second SRTM DEM

Refined stream height setting for adjacent SRTM grid tiles



Hutchinson et al 2009 *MODSIM*
Hutchinson et al 2011 *Geomorphometry*

Vector field interpretation of specific catchment area

Assign the downslope direction to specific catchment area $\boldsymbol{\rho}$
$\boldsymbol{\rho}$ becomes a 2-dimensional vector field satisfying

$$div\ \boldsymbol{\rho} = 1$$

by the integral definition of *div*

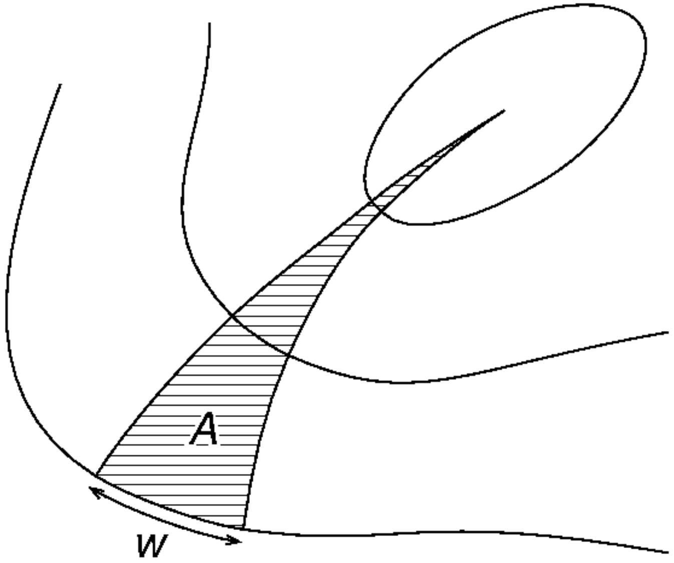
Using orthogonal coordinates of contours *u* and flow lines *v* it immediately follows that

$$(1/h_u h_v)\ \partial a h_u/\partial v = 1$$

So as in Gallant & Hutchinson (2011)

$$\partial a/\partial l = 1 - a.\kappa_c$$

where l = flow line length and κ_c = contour curvature



$$a = \lim_{w \to 0} \frac{A}{w}$$

Issues for vector field ρ

- Not a potential field in general - so no simple shortcuts to its calculation
- Integrals of ρ around grid cell boundaries are well defined and could be one way to overcome the singularities in ρ itself
- An optimum method for calculating such integrals is still being contemplated – since the 1980s!

Conclusion

- Process basis can lead to enduring models
- Appropriate mathematics
- Topographic scale, and hierarchies of scale, are important
- No model is perfect – ANUDEM continues to evolve
- Details – outliers, remaining sinks, etc – matter
- User testing and experience matter
- Greatly indebted to my immediate colleagues as well as constructive feedback from users around the world