15

43

# Scale-effect of Hypsometric Integral in Loess Plateau

Zhu Shijie
Key Laboratory of Virtual Geographic Environment, Ministry
of Education
Nanjing Normal University
Nanjing, China
zsjsdx1025@163.com

16 Abstract—The Loess Plateau is gully and hilly with serious soil 17 erosion and thus a thorough study on the landscape development 18 and erosion process with the scope of small watershed is of vital 19 importance for ecological restoration of this region. As a 20 macroscopic landscape analyzing method, hypsometric integral 21 could be used for a comprehensive analysis on the development 22 stages and process of small watersheds. Nevertheless, this method 23 could produce distinct scale-effect and consequently an in-depth 24 recognition on the finest analysis scale is a basis for properly 25 understanding the characteristics and development of the 26 watershed. This paper, with small watershed in Loess Plateau as 27 the object of study, elaborately analyzes the scale-effect of 28 hypsometric integral from the perspectives of analysis scale, 29 statistical scale and regional scale based on 5-meter-resolution 30 DEMs. The results show that, with the increase of hypsometric 31 series under analytical scale, the integral value decreases sharply to 32 close to the true value, yet the decreasing range narrows greatly 33 and finally hypsometric curve becomes more accurate. As DEM 34 resolution in statistical scale rises, hypsometric integral value 35 basically remains unchanged, and this value is closely related to the 36 other two land surface parameters, depth of surface cutting and 37 relief height of landscape. Moreover, when considering regional 38 scale in the same landscape, the smaller the analytical area is, the 39 greater the integral value difference will present. At last, the 40 integral value tends to remain steady when analytical area reaches 41 a certain degree. This paper lays a foundation for further analysis 42 of watershed characteristics.

### I. INTRODUCTION

<sup>44</sup> Hypsometric integral is a terrain analysis factor with apparent <sup>45</sup> physical and geomorphologic meanings which could reflect the <sup>46</sup> landform erode stage and evolution process. As a macroscopic <sup>47</sup> parameter and method in terrain analysis, the applications of <sup>48</sup> hypsometric integral could reveal the quantitative characteristic <sup>49</sup> of landform evolution in watershed scale. However, it still needs <sup>50</sup> to be improved and enriched of the hypsometric integral analysis <sup>51</sup> method, particularly in the little progress has been made of <sup>52</sup> hypsometric integrals in the applications of watershed evolution <sup>53</sup> and geomorphologic pattern from previous literatures.

8	Tang Guoan
9	Key Laboratory of Virtual Geographic Environment, Ministry
10	of Education
11	Nanjing Normal University
12	Nanjing, China
13	tangguoan@njnu.edu.cn

Horton firstly put forward quantitative indicators to describe 54 55 watershed morphology in 1932, followed by Strahler (1952, 1963) 56 and other geomorphologists who constantly improved and 57 complemented the quantitative description indicator system of 58 the watershed (Schumm, 1956; Morisawa, 1964; Lu, 1991). Chen 59 and Jiang (1986) summerized quantitative interrelations between 60 watershed quantitative description indicator system and factors 61 involved in their book Watershed Geomorphologic Mathematic 62 Model. Hypsometric integral, initiated by American 63 geomorphologist Strahler as early in 1952, mainly studied the 64 relationship between regional horizontal section area and 65 hypsometry, which contributes a great deal to the quantitative 66 Davis Theory of Geomorphological Cycle. And later, scholars at 67 home and abroad made further studies on it. Willgoose (1998) 68 analyzed how the watershed morphology and drainage density 69 influenced hypsometric integral. Moreover, Masek (1994) and 70 Montgomery (2001) concluded that different sediments and 71 runoffs had a great impact on hypsometric integral. Chen (2005) 72 analyzed the variations of hypsometric integral from the aspect of 73 drainage area and spatial distribution. Walcott (2008) illustrated 74 specifically the spatial distribution of hypsometric integral in 75 different scales with watershed in southeastern Africa as study 76 object. With respect to the application of losses geomorphy, Wu 77 (1965) comprehensively explained hypsometric integral's 78 application in geography and pointed out its insufficiency mainly 79 related to the conceptual feature of the curve. He held that it 80 could only be used to summarize the characteristics of landform 81 morphology. Besides, Ai (1987, 1988) proposed the comentropy 82 of the watershed erosion system based on hypsometric integral 83 and introduced the concept of linear unbalanced comentropy into 84 the study of watershed system. Li and Lu (1990) developed 85 erosion integral on the basis of hypsometric integral and analyzed 86 the characteristics of watershed development stages from the 87 angle of erosion. In addition, Xin (2008) made a spatial chart of <sup>88</sup> hypsometric integral in losses plateau and drew the conclusion 89 that with the variance of digital resolution, HI value, short for <sup>90</sup> hypsometric integral, remained steady (100m as a starting point), 91 while as analytical window increases gradually, HI value 92 decreases by power function. Liao (2008) held that hypsometric

### Geomorphometry.org/2013

93 integral could reflect present landform status and erosive 125 94 tendency of the watershed, and the size and changing trend were 95 closely related to whether watershed bottom reached basement.

97 good comprehension on it is the key to providing the basis for 129 analyzes the influence produced by hypsometric classification in 98 thorough understanding of geographical characteristics and 130 hypsometric integral calculation and curve plotting, namely 99 geographical phenomenon process. DEM itself depends so 131 analytical scale. Then, with respect to DEM resolution, a 100 heavily on scale that scale-effect is absolutely inevitable while 132 discussion is made here on variance of hypsometric integral 101 using DEM to calculate hypsometric integral value. Liu (2007) 133 value with different resolution. Finally, it works out hypsometric 102 summarized the scale-effect of DEM, analyzed DEM scale 134 integral of sub-catchments differing in watershed area within the 103 conceptual model and scale system, and argued that scale could 135 small watershed and discusses the issues on regional scale of 104 be classified into two groups, namely, geographical scale and 136 hypsometric integral. 105 representation scale with the latter including analytical scale, <sup>105</sup> representation scale what the latter is a paper, with typical small <sup>137</sup> 107 watershed in loess plateau as a study case and DEM data as basic 108 data source, analyzes basic characteristics of hypsometric integral 138 109 within small watershed and explores the influence on 110 hypsometric integral posed by analytical scale, digital scale and <sup>111</sup> regional scale contained in application of hypsometric integral.

### II . DATA AND METHODS

#### A. Sample Region and Data 113

112

124

114 115 soil erosion is serious. This paper concentrates on severely 147 series is n and then elevation in each level is  $\Delta h/n$ . The number 116 erosive area in loess plateau and selects typical types of loess 148 of frequency value we can get from elevation frequency 117 landforms with three regions respectively loess tableland, loess 149 histogram is n. Based on corresponding frequency value and 118 ridge and loess hill (Fig.1). Then, based on watershed 150 cumulative area; the hypsometric curve can be drawn. While 119 completeness and data accessibility, taking 1:10000 DEM of 151 calculating, we set aside classification series-an input parameter, 120 national fundamental geographic data as basic data, it extracts 152 for different series could show how classification affects plotting 121 three small watersheds and analyzes integrated characteristics 153 of the curve and calculating of integral value. 122 and scale-effect of its hypsometric integral.



Figure 1. Distribution of test areas on the Loess Plateau

### B. Method

The paper takes ArcGIS9.3 as a processing platform for sub-127 catchment division and DEM re-sampling and programs with As scale-effect widely existing in geo-analysis, therefore, a 128 MatLab software to calculate hypsometric integral. Firstly, it

### **III. SCALE-EFFECT ANALYSIS**

### A. Classification Effect

Hypsometric integral mainly comprises two parts, namely, 139 140 the integral value itself embodies and the hypsometric curve 141 plotted according to elevation cumulative frequency. While 142 plotting the curve, one should find out a series of points based on 143 which pattern of the curve will be drafted. The classification 144 series equals to curve points. Specifically speaking, it means the 145 number of elevation classification of the studying area, that is to The Loess Plateau is covered with gully and hilly areas and  $_{146}$  say, suppose regional elevation difference is  $\Delta$ h, classification

> According to the method used to calculate hypsometric 154 155 integral, the factor that mainly influences its integral value and 156 hypsometric curve is the percentage each elevation accounts in 157 all classification of elevation. From definition of hypsometric <sup>158</sup> integral, integral value can only be true when classification series 159 is infinite. While in practice, we can only calculate percentage of 160 area of each level and plot the curve by fixing a certain 161 classification number. The more it classifies, the more precise the 162 integral value is, thus bringing a more close-to-reality curve. 163 However, when classification series increases, efficiency of 164 calculation decreases straightly (Fig.2). In this case, a balance is 165 needed which can ensure both accuracy of integral value and 166 efficiency of curve-plotting. It's shown by difference in adjacent 167 integral value, known as its decreasing range. As classification <sup>168</sup> number increases, the decreasing scope of integral value narrows 169 accordingly. And when it reaches 100-level, the value decreases 170 is smaller than 0.0005. Taken overall consideration of integral <sup>171</sup> value and curve, the classification series set in this paper is 100.



172 173 174

### Figure 2. HI against the number of classification

#### B. Resolution Effect 175

Analysis on resolution dependence mainly focuses on its 176 177 analytical characteristics of hypsometric integral under different 214 in the area analyzed. 178 resolution DEM data so as to explore the resolution sensitivity 179 Taken 5-meter resolution and scale of 1:10000 DEM as basic 215 180 data source, resample the three small watersheds by using 181 quadratic linear interpolation algorithm with ReSample tool in 182 ArcToolBox, and set resolution interval as 10, and then we can 183 get 19 resolution data ranging from 5 to 195m respectively. Next, 184 we could explore its resolution dependence by calculating 185 hypsometric integral of each watershed in different series of 186 resolution scale (Fig.3). As a smaller resolution can reflect relief 187 characteristics of landforms better, therefore, this paper supposes 188 the 5-meter DEM data closes to real landform data.



#### Figure 3. HI against Resolutions 190

191 192 resolution in the three watersheds, as shown in the figure, 237 Compared with higher reaches of the gully, that is gully head 193 hypsometric integral value of each watershed remains steady 238 areas, the erosion here is relatively weaker and the integral value 194 basically, and maintains constant with resolution changing. From 239 is lower accordingly, which to some extent shows this type of 195 the definition of hypsometric integral, physically it represents, 240 watersheds maintain stable and stay in their later stage of erosion. 196 compared with erosion basis, the ratio of potential erosive 241 Thus, same as the watershed classification, this type of sub-197 amount of the remaining watershed development erosion area.

$$HI = \frac{V_1}{V_0}$$

189

200 erosion basis within watershed, while Va indicates the volume of 246 sub-watershed is as follows (Fig. 4), from which we can see that 201 all materials above erosion basis with no matter run off.

KeyAuthor et al

Taken DEM volume calculation as an example, 202

 $Vr = h_msam \times g^2 * n - h_min \times g^2 \times n = (h_msam - h_min) \times g^2 \times n$ 

 $Va = h_max \times g^2 * n - h_min \times g^2 \times n = (h_max - h_min) \times g^2 \times n$ Where, g is grid length,  $g^2$  means grid area, and n shows the 205 206 number of grids the watershed occupies.

Dividing one by the other results in: 207

$$HI = \frac{(h\_meam - h\_min) \times g^2 \times n}{(h\_max - h\_min) \times g^2 \times n} = \frac{h\_meam - h\_min}{h\_max - h\_min}$$

The numerator is cutting depth of surface and denominator is 209 210 relief amplitude. Pike (1971) confirmed in his essay the 211 correctness of the formula from angle of infinitesimal calculus. In 212 this regard, the variance of hypsometric integral value is 213 determined mainly by elevation extremum and average elevation

### C. Watershed Size Effect

208

216 Hypsometric integral is a regional macro-statistical indicator 217 analyzing landform characteristics within the region. Obviously, 218 analysis results are inevitably diverse with different regions. How 219 to ensure the stability of hypsometric integral under various 220 landforms is a problem we must deal with. Meanwhile, 221 hypsometric integral also serves as an indicator to show 222 development stage of watershed and rectangular window analysis 223 method by no means can summarize its basic characteristics 224 completely. Therefore, the paper, in analysis of integral <sup>225</sup> dependence, calculates integral value by using watershed analysis 226 window with whole watershed as its object. The details are as 227 follows: first, divide small watershed into several sub-watersheds 228 by threshold value, and grant such sub-watersheds corresponding 229 classification according to Strahler's classification and level of 230 catchment network. Then, draw complete sub-watershed in 231 accordance with classification and calculate hypsometric integral 232 value. At the same time, in sub-watershed division, certain inter-233 areas exist which have a water inlet thus differing from complete 234 watershed. The boundaries of such inter-areas aren't watershed in 235 strict sense due to the existence of the inlet. The inter-areas lie in Calculate respectively the hypsometric integral of different 236 middle and lower reaches of gully that are traversing across. 242 watershed is not considered here.

Three levels of several sub-watersheds can be gained by 243 <sup>244</sup> dividing three small watersheds, hypsometric integral value can Vr represents the volume of remained materials beyond 245 thus be figured out. The scatter diagram on the integral value and 247 the range of integral value is wide while the area of sub-<sup>248</sup> watershed is small. With the increasing area of sub-watersheds,

### Geomorphometry.org/2013

249 its integral value has a trend of convergence. The terrain feature 293 Foundation of State Key Laboratory of Resources and 250 can be displayed by a series of the combination of landform 294 Environmental Information System (No. 2010KF0002SA). 251 characteristic, which absolutely occupy some space. When its 252 area is smaller than a certain value (stable area), the 295 253 geomorphologic feature of the landform types cannot be <sup>254</sup> completely displayed, which will increase the uncertainty of the 255 indicators reflecting the terrain feature. Just like the stability of 298  $_{256}$  the slope spectrum (Tang, 2008), the hypsometric integral, as a  $_{299}^{270}$ 257 statistical indicator, also has the inner characteristic of area 300 258 stability. 301



### 260

## Figure 4. HI against the Watershed Size

261

290

### IV CONCLUSION

The hypsometric integral value represents a function related 314 262 263 to surface cutting depth and relief amplitude, but it is unrelated to 315 264 its own classification series and DEM resolution. However, it is 316 265 essential to choose the classification series in drawing the 317 266 hypsometric curve. With the increase of classification, the 318 267 integral value figured out on the basis of different levels has a 319 268 trend of gradual decrease with sharp amplitude. That is, when the 320  $_{269}$  classification reaches a certain level, the magnitude is less than  $_{321}^{321}$  $_{270}$  five ten thousandth as the integral value only changed very  $_{323}$  $_{271}$  slightly. As a result, the influence on the integral value can be  $_{324}^{325}$ 272 ignored. Similarly, due to the characteristic of hypsometric 325 273 integral, basically it has the trend of constant arrangement in the 326 274 calculation of its value with different DEM resolution. 327 275 However, it also has a certain fluctuation, which is much 328 276 consistent to the maximum or minimum fluctuations of DEM 329 277 resolution. This is mainly because the re-sampling method, which, 330 278 to some extent, has changed the hypsometric extreme value of 331 279 DEM. When the precision of hypsometric integral is guaranteed, 332  $_{280}$  it is reliable to determine the value based on coarse DEM  $_{333}^{333}$  $^{334}_{281}$  resolution. When the areas of sub-watersheds are small, the  $^{334}_{335}$  $_{282}$  differences among the hypsometric integral values are very large  $_{336}^{_{330}}$ <sup>283</sup> in the same type of landforms, mainly because the local area of <sup>337</sup> 284 sub-watersheds is too small to represent and completely reflect 338 285 the terrain feature of its landform. Consequently, it is necessary 339 286 to consider its area dependence in order to reveal the 340 287 development laws of characteristics of watersheds in the analysis 341 288 of regional difference of landforms using the hypsometric 342 343 289 integral method. 344

### ACKNOWLEDGMENT

This research is supported by National Natural Science 291 292 Foundation of China (No. 40930531, 41071188); Open

4

345

309

310

311

312

313

KeyAuthor et al.

### REFERENCES

[1] Horton, R.E. 1932. "Drainage basin characteristics". Transactions of the American Geophysical Union.

[2] Strahler, A.N. 1952. "Hypsometric (area-altitude) analysis of erosional topography". Geological Society of America Bulletin, 63, 1117-1142

Strahler, A.N. 1963. "The earth sciences, Harper and Row". Pub New [3] York and London.

[4] Schumm, S.A. 1956. "The role of creep and rainwash on the retreat of Badland slopes". Amer. Jour. Sci., 254: 693-706.

[5] Morisawa, M.E. 1964. "Development of drainage systems on an upraised lake floor". Amer. Jour. of Science. 262, 341-354.

[6] Lu, Z.C., Jia, S.F., Huang, K.X., Yuan, B.Y. 1991. "Watershed Landform System". Dalian: Dalian Press (in Chinese).

Cheng, J.C., Jiang, M.Q.1986. "Digital Model of Watershed Landform". Beijing: Science Press (in Chinese).

Willgoose, G., Hancock, G. 1998. "Revisiting the hypsometric curve [8] as an indicator of form and process in transport limited catchments". Earth Surface Processes and Landforms, 23: 611-623.

[9] Masek, J.G., Isacks, B.L., Gubbels, T.L., Fielding, E.J. 1994. "Erosion and tectonics at the margins of continental plateaus." Journal of Geophysical Research, 99: 13941-13956.

[10] Montgomery, D.R., Balco, G., Willet, S.D. 2001. "Climate, tectonics, and the morphology of the Andes". Geology, 29: 579-582.

[11] Chen, Y.J., Zheng, G.Y., Song, G.C. 2005. Influence of Area and Space Dependence for Hypsometric Integral and its Geological Implications, journal of Geographical Science (Taiwan), 39:53-69. (in Chinese)

[12] Walcott Rachel C. Summerfield M.A. 2008. "Scale dependence of hypsometric integrals: An analysis of southeast African basins". Geomorphology. 96:174-186.

[13] Wu, H.H. 1965. "Application of Hypsometric curve". Acta Geographica Sinica, 31(2):157-169. (in Chinese)

[14] Ai, N.S. 1987. "Comentropy in Erosional Drainage-System". Acta Conservations Soil. 1(2):1-8. (in Chinese)

[15] Ai, N.S., Yue, T.X. 1988. "Second Discussion of the Comentropy of Drainage-System." Acta Conservations Soil. 2(4):1-9. (in Chinese)

[16] Xin, Z.B., Xu, J.X., Ma, Y.X. 2008. "Hypsometric Integral Analysis and its Sediment Yield Implications in the Loess Plateau, China." Journal of Mountain Science. 26(3):356-363. (in Chinese)

[17] Li, Q., Lu, Z.C., Yuan, B.Y. 1990. "Quantitative Study of the Stage of Geomorphplogical Evolution." Acta Geographica Sinaca. 45(1): 110-120. (in Chinese)

[18] Liao, Y.S., Cai, Q.G., Qin, F., Zhang, J.B., Ding, S.W. 2007. "Study on Topographic Evolution and the Eroding Trend in Hilly Loess Areas, North China." Journal of Mountain Science. 26(3):347-355. (in Chinese)

[19] Liu, X.J., Lu, H.X. 2007. "Scale Issues in Digital Terrain Analysis and Terrain Modeling." Geographical Research. 26(3): 433-442.

[20] Tang, G.A., Li, F.Y., Liu, X.J. Research on the Slope Spectrum of the Loess Plateau. Science in China Series E: Technological Sciences, 2008, 51(Supp.1): 175-185.