Extend the Analysis window to improve the Geo-Computation

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Many Geo-Science models demand computation of partial derivative, such as slope/aspect model, kinds of curvature model, and the HASM. It is quite obvious that the accuracy of the partial derivative could affect the last computation result. Nowadays, the basic difference scheme to compute partial derivative is central difference, which is simple but the accuracy is also low. So it is important to find new difference scheme that could afford higher accuracy and don't demand more memory and more solution time.

In order to construct the demanded difference scheme, firstly we analyze the error source during the computation of the partial derivative. Take the central difference as example, the computation of central difference is based on 3×3 window (Fig 1).

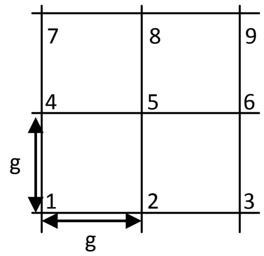

Fig.1 3×3 local moving window

Suppose we have to compute the partial derivative at point 5, considered the Taylor expansion:

$$f\left(x+g,y\right)=f\left(x,y\right)+f_x\left(x,y\right)g+f_x^{''}\left(x,y\right)g^2/2+f_x^{'''}\left(\xi_x,y\right)g^3/3! \qquad (1)$$

$$f\left(x-g,y\right)=f\left(x,y\right)-f_x\left(x,y\right)g+f_x^{''}\left(x,y\right)g^2/2-f_x^{'''}\left(\gamma_x,y\right)g^3/3! \qquad (2)$$

while， $\xi_x\in\left(x,x+g\right)$， $\gamma_x\in\left(x,x-g\right)$； f_x， $f_x^{''}$， $f_x^{'''}$ are respectively the first, the second, and the third partial derivative. Now we make the (1) subtract the (2), the error of the partial derivative at X direction could be represented as:

$$Error_x=\left(f\left(x+g,y\right)-f\left(x-g,y\right)\right)/2g-f_x\left(x,y\right)$$

$$=g^2\left[f_x^{'''}\left(\xi_x,y\right)+f_x^{'''}\left(\gamma_x,y\right)\right]/\left(2.3!\right) \qquad (3)$$

Similarly, the error of the partial derivative at Y direction could be represented as：

$$Error_y=\left(f\left(x,y+g\right)-f\left(x,y-g\right)\right)/2g-f_y\left(x,y\right)$$

$$=g^2\left[f_y^{'''}\left(x,\xi_y\right)+f_y^{'''}\left(x,\gamma_y\right)\right]/\left(2.3!\right) \qquad (4)$$

while, $\xi_y \in \left(y, y+g\right)$, $\gamma_y \in \left(y, y-g\right)$; f_y, $f_y^{'''}$ are respectively the first, the second, and the third partial derivative.

Through (3) and (4), it is obvious that the partial derivative error is directly proportional with the window size g^2. So the error could be decreased if the window size is cut off. Suppose the new window size is half of the former, then we could get the new presentation of the error of partial derivative at X direction and Y direction:

$$Error_x^{'}=\left(f\left(x+g/2,y\right)-f\left(x-g/2,y\right)\right)/2.(g/2)-f_x\left(x,y\right)$$

$$=\left(g/2\right)^2\left[f_x^{'''}\left(\bar{\xi}_x,y\right)+f_x^{'''}\left(\bar{\gamma}_x,y\right)\right]/\left(2*3!\right) \qquad (5)$$

$$Error_y^{'}=\left(f\left(x,y+g/2\right)-f\left(x,y-g/2\right)\right)/2\left(g/2\right)-f_y\left(x,y\right)$$

$$=\left(g/2\right)^2\left[f_y^{'''}\left(x,\bar{\xi}_y\right)+f_y^{'''}\left(x,\bar{\gamma}_y\right)\right]/\left(2*3!\right) \qquad (6)$$

while, $\bar{\xi}_x \in \left(x, x+g/2\right)$, $\bar{\gamma}_x \in \left(x, x-g/2\right)$, $\bar{\xi}_y \in \left(y, y+g/2\right)$, $\bar{\gamma}_y \in \left(y, y-g/2\right)$。

Suppose the window size is small enough, there are $f_x^{'''}\left(\xi_x,y\right)\approx f_x^{'''}\left(\bar{\xi}_x,y\right)$,

$f_x^{'''}\left(\gamma_x,y\right)\approx f_x^{'''}\left(\bar{\gamma}_x,y\right)$, $f_y^{'''}\left(x,\xi_y\right)\approx f_y^{'''}\left(x,\bar{\xi}_y\right)$, $f_y^{'''}\left(x,\lambda_y\right)\approx f_y^{'''}\left(x,\bar{\gamma}_y\right)$.

Then we could get the following equations:

$$\frac{Error_x^{'}}{Error_x}=\frac{1}{4},\quad \frac{Error_y^{'}}{Error_y}=\frac{1}{4} \qquad (7)$$

Combine the (5)、(6)、(7)、(8) and (9), the new partial derivative at X direction and Y direction could be represented as following:

$$f_x\left(x,y\right)=\frac{1}{3}\left[4\times\left(\left(f\left(x+g/2,y\right)-f\left(x-g/2,y\right)\right)/2\left(g/2\right)\right)-\left(f\left(x+g,y\right)-f\left(x-g,y\right)\right)/2g\right] \qquad (8)$$

$$f_y\left(x,y\right)=\frac{1}{3}\left[4\times\left(\left(f\left(x,y+g/2\right)-f\left(x,y-g/2\right)\right)/2\left(g/2\right)\right)-\left(f\left(x,y+g\right)-f\left(x,y-g\right)\right)/2g\right] \qquad (9)$$

Since cut off the window size means change the resolution of the origin data such as DEMs, so we construct the new partial derivative model through extending the analysis window (Fig 2). Suppose we compute the partial derivative at point 13, then the partial derivative at X direction and Y direction could be represented as following:

$$\begin{cases} f_x = \frac{1}{3}\left[4\times(z_{18}-z_8)/2g-(z_{23}-z_3)/4g\right] \\ f_y = \frac{1}{3}\left[4\times(z_{14}-z_{12})/2g-(z_{15}-z_{11})/4g\right] \end{cases} \tag{10}$$

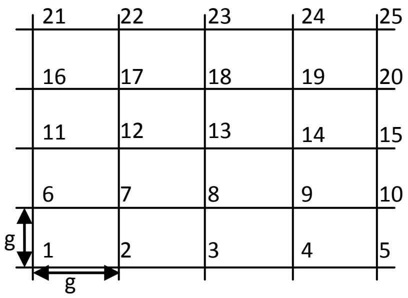

Fig.2　5×5 local moving window

We take slope calculation as example to test the accuracy of the new partial derivative computation model. Firstly, a mathematical surface (Fig 3) was selected to test the accuracy of the new model because the mathematical could afford the real slope value.

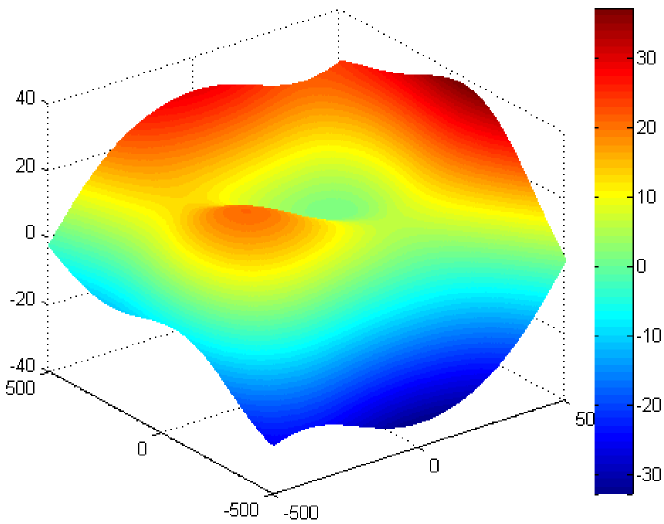

Fig 3 mathematical surface

The equation of the mathematical surface is:

$$f_1(x,y)=10\left[3\left[1-\left(\frac{x}{500}\right)^2\right]e^{-\left(\frac{x}{500}\right)^2-\left(\frac{y}{500}+1\right)^2}-10\left[0.2\left(\frac{x}{500}\right)-\left(\frac{x}{500}\right)^3-\left(\frac{y}{500}\right)^5\right]e^{-\left(\frac{x}{500}\right)^2-\left(\frac{y}{500}\right)^2}-\frac{1}{3}e^{-\left(\frac{x}{500}+1\right)^2-\left(\frac{y}{500}\right)^2}\right]$$

Two kinds of DEMs with different resolution were generated based on the mathematical surface, one is 1m, and the other is 5m. Then slope was computed respectively use two slope models, one is based on central difference, and the other is based on the new partial derivative difference scheme put forward in this paper. Table 1 shows the accuracy analysis result (RMSE of the calculation result) of the slope calculation.

Tab.1 Error analysis of slope

DEM resolution	Central difference	New difference scheme
1m	3.40×10^{-5}	4.46×10^{-10}
5m	8.41×10^{-4}	2.77×10^{-7}

The results show that the new model can significantly improve the accuracy of the result compared with the common models. This study enriches the analysis system of the digital terrain analysis system, and provides slope data of high accuracy for many Geo-science models. In addition, except slope, there are many terrain parameters which are calculated through finite difference, such as aspect and various kinds of curvature, and the methods of this paper could afford some useful references in improving the accuracy of such terrain parameters.

Reference:

[1] LIIU Xuejun. On the Accuracy of the Algorithms for Interpreting Grid-based Digital Terrain Model [D]. Wuhan: Wuhan University, 2002. (In Chinese)

[2] Fleming, M.D., Hoffer, R.M., 1979, Machine processing of Landsat MSS data and DMA topographic data for forest cover type mapping: West Lafayette, IN, Purdue, University, Laboratory for Applications of Remote Sensing, LARS Technical Report 062879.

[3] Unwin. 1981, *Introductory Spatial Analysis*, Methuen, London and New York.

[4] Sharpnack, D.A., Akin, G., 1969, An Algorithm for computing slope and aspect from elevations, *Photogrammetric Survey*, 35: 247-248.

[5] Horn, B.K.P. 1981, Hill shading and the reflectance map, *Proceedings of IEEE*, 69(1): 14-47.

[6] ZHOU Qi-ming, LIU Xue-jun. Analysis of errors of derived slope and aspect related to DEM data properties [J]. Computer & Geosciences, 2004, 30:369-378.

[7] Hodgson, M.E., 1995, What cell size does the computed slope/aspect angle represent? *Photogrammetric Engineering and Remote Sensing,* 61: 513-517.

[8] Jones, K.H., 1998, A comparison of algorithms used to compute hill slope as a property of the DEM, *Computer and Geosciences*, 24(4): 315-323.

[9] Carter, J.R., 1992, The effect of data precision on the calculation of slope and aspect using gridded DEMs, *Cartographica*, 29(1): 22-34.