

# *Generalizing Resolution-assignable DEM Based on Rational-dilation Wavelet Transform and Sampling Theory*

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**Abstract:** The digital elevation model (DEM) is widely used in various geographic researches and investigations. However, the scale-effect of the model deeply impacts the suitability and reliability of its application. The studies on the scale-effect of DEM thus have becoming focuses of researches. As these studies require the DEM dataset with continuously-changeable resolutions and uniformly spatial-location base in analysis and evaluation and the present method can not scientifically and systematically meet the required, we propose an effective DEM generalization method to obtain lower-resolution DEM based on higher-resolution DEM.

The method is based on the sampling theory and a kind of wavelet transform. The relation between the horizontal resolution of DEM and its max frequency in frequency domain is firstly analyzed according to the sampling theory which is used in signal resampling as a fundamental principle. We then propose a new method to obtain the DEM with assignable resolution by quantitatively decreasing the highest frequency of the original DEM. After studying the idea of this proposed method, we improve a kind of rational-dilation wavelet transform (RWT) to meet the requirement of the proposed DEM generation method. The original RWT, whose Q-factor can be tuned before decomposition, has a finer time-frequency localization ability than common dyadic wavelet transform. However, its fixing employment of Q-factor in all decomposition level greatly restricts its decomposition freedom in frequency-domain partition, and also the up-sampler and down-sampler in its decomposition destroys its translation-invariant property. We analyze the decomposition and reconstruction of the original RWT, and then construct a new transform scheme that provides a tunable Q-factor in each level and keeps the translation-invariant property as well. Also, this new RWT method is expanded to two-dimensional case by using tensor-product transform approach for image signal processing. After studying and evaluating the freely and finely time-frequency decomposition ability of the constructed RWT, we propose an energy-based threshold processing method used in the decomposition domain of the RWT. The method implements threshold processing to different

frequency-band obtained from the decomposition of the RWT. With the method, the most significant spatial-domain features in the original data are preserved and at the same time, it is ensured that the DEM data obtained can meet the requirement of the sampling theory. The generalization method to generate resolution-assignable DEM is finally obtained.

With the DEM generation method, from a high-resolution DEM, we can obtain a DEM dataset of which the resolutions change from fine to coarse. Moreover, the validity of the assigned resolution, the spatial location consistency and the effectiveness of generalization are all analyzed and evaluated. The result shows the resolution of the obtained DEM are all satisfied with the sampling theory and the ratio of their sampling rate to their max frequency are all very close to that of original high-resolution DEM. Those show the reasonability of the resolution of the obtained DEM dataset. To evaluating the spatial location consistency, we analyze the contour overlay status of the DEM dataset obtained by using the proposed method, and the result shows the overlay status is more reasonable than those of the DEM obtained based on interpolation of multi-scalar topographic map. The generalization effect is evaluated by visually comparing, analyzing hillside images, elevation statistics, slope frequency distribution curves and cumulative slope frequency distribution curves, curvature frequency distribution curves and cumulative curvature frequency distribution curves, total contour lengths and total river network length. The evaluation results show that as the resolution of the obtained DEM changes from finer to coarser, the local detail of terrain is gradually eliminated, while in this process, the overall structure of the terrain is still retained. We even compare our generated DEM data of different resolution with DEM data obtained by interpolation of topographic map, and find that for a certain resolution, the DEM obtained by our method have similar terrain characteristics (including visual observation, elevation statistics, slope frequency distribution curves and cumulative slope frequency distribution curves, curvature frequency distribution curves and cumulative curvature frequency distribution curves, total contour lengths and total river network length) as that of

topographic map interpolation. In addition, we evaluate the generation error of the obtained DEM by analyzing their mean square error. We find the results generalized have desirable generation precision and in most case, their generation errors are lower than those of the interpolated DEM obtained by using conventional interpolation methods (e.g., bilinear interpolation and bicubic interpolation).

In this paper, we study the relation between the spatial horizontal resolution of DEM and its effective max-frequency in frequency domain based on sampling theory. We proposed a DEM generalization method by which one can obtain DEM data with coarse and assigned resolution from original DEM with fine resolution. Our experiment results show that with the method proposed, one can obtain the DEM data which has the goal resolution assigned in advance. Also, the generalization effect of method is reasonable, the spatial-location base consistency of the DEM dataset and the generalization precision is more desirable than other conventional methods. This study provides methodological supports to digital terrain analysis, researches and investigations.

*Key words:* DEM, generalization, Rational-dilation wavelet transform, Resolution, Q-factor