# Adaptive smoothing for noisy DEMs

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*Abstract*—**DEMs derived from dense remotely sensed measurements, including lidar- and radar-based DEMs, provide much greater surface detail than traditional interpolated DEMs but suffer from random noise that perturbs measures of surface shape such as slope and flow direction. Smoothing is an effective method of reducing noise but also tends to impact on important surface features, lowering hilltops, raising valleys and obliterating important fine detail. This paper describes a multiscale adaptive smoothing approach that responds to both the relief and noise level in a DEM by smoothing aggressively where the noise is larger than the local relief and smoothing little or not at all where noise is less than relief. The method is simple and efficient and can be readily implemented in a raster GIS environment. The method is demonstrated on noisy SRTM data.**

## INTRODUCTION

A digital elevation model (DEM) is an imperfect representation of a real land surface. The impact for geomorphometric applications of the imperfections in the DEM depends on how they affect measures of surface shape such as slope, flow direction and curvature.

Through the formative years of computerized geomorphometry, or digital terrain analysis, most DEMs were produced by interpolation of relatively sparse source data mostly derived from topographic maps. Such DEMs are locally smooth and the main source of imperfection was the lack of detail in the surface form, particularly in low relief areas where contours are widely spaced.

More recently many DEMs are produced from dense remotely sensed measurements by radar, lidar or photogrammetric methods. These DEMs typically have at least one measurement for every grid cell so capture surface detail well but the measurements are usually subject to error. This appears as noise in the elevation data, with varying characteristics depending on the data source. Measures of shape that depend on local differences in elevation are severely affected by random noise and tend to be more of a problem in low relief areas where shapes are subtle.

Smoothing by local averaging is an effective procedure for reducing noise but involves trading off the level of smoothing against the preservation of real terrain features: a large smoothing kernel removes noise well but tends to obliterate small features and rounds sharp edges, while small kernels preserve the terrain features but do not remove noise effectively. Ideally a smoothing method should provide more smoothing where noise is large relative to the topographic variation and little or no smoothing where the noise is much smaller than the topographic variation. The different signal-to-noise ratios can be due to both varying signal levels (topographic variation) and to varying noise levels.

This paper describes such an adaptive smoothing method that removes noise while preserving terrain features, responds to varying noise levels and can also fill in missing data. It uses a multi-resolution statistical approach that is efficient and can be readily implemented in a raster GIS environment.

## THE ADAPTIVE SMOOTHING METHOD

The adaptive smoothing method is based on the ideas of Lee [1] but extended to multiple resolutions. Lee's method computes the local mean $\bar{z}_{i,j}$ and noise-adjusted variance $Q_{i,j}$ at location $i, j$ then derives an estimated value as a weighted sum of the local mean and the original noise-corrupted value:

$$\hat{x}_{i,j} = \frac{\sigma_1^2}{Q_{i,j} + \sigma_1^2}\bar{z}_{i,j} + \frac{Q_{i,j}}{Q_{i,j} + \sigma_1^2} z_{i,j} \qquad (1)$$

where $\sigma_1^2$ is the variance of the noise; $x_{i,j}$ denotes the actual value at $i, j$ and $z_{i,j}$ the noise-corrupted value. The effect is that where the variation in the noisy signal is significantly larger than the noise, the noisy value is used as estimated actual value since the noise does not have a big impact; where the variation is small compared to the noise the local mean is used, reducing the noise



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significantly. Lee's method has been used to smooth DEMs, for example Simard et al [2] who used a Lee-type filter at 5x5 to smooth SRTM elevations with a fixed noise standard deviation of 1.8m

Lee notes that "The use of different window sizes will greatly affect the quality of processed images. If the window is too small, the noise filtering algorithm is not effective. If the window is too large, subtle details of the image will be lost in the filtering process." The solution to the choice of window size in this adaptive smoothing method is to smooth over multiple window sizes, letting the variance at each window size control how much the mean at that window size contributes to the estimated value.

The algorithm accounts for spatially varying noise variance and computes the means and variances on nested windows so that all calculations after the first resolution step are performed on progressively coarser grids, leading to very efficient processing.

The multi-resolution algorithm is similar to a multi-scale Kalman smoothing method [3,4] and consists of a series of progressive aggregations followed by a series of refinements back to the original resolution.

The algorithm is initialized with:

$$\bar{z}^0 = z,\ w^0 = \frac{1}{v^0},\ w_{sq}^0 = \left(w^0\right)^2,\ v_g^0 = 0,\ n^0 = 1 \qquad (2)$$

except at locations with no data which are initialized with:

$$w^0 = 0,\ w_{sq}^0 = 0,\ v_g^0 = 0,\ n^0 = 0 \qquad (3)$$

$w$ is the weighting for each cell, equal to the inverse of variance $v$, and $n$ is the number of cells with data.

Then for each step $i$ from 1 to $i_{\max}$:

$$w^i = \sum w^{i-1} \qquad (4)$$

$$w_{sq}^i = \sum w_{sq}^{i-1} \qquad (5)$$

$$\bar{z}^i = \frac{\sum w^{i-1}\bar{z}^{i-1}}{w^i} \qquad (6)$$

$$v_{bg}^i = \frac{\sum w^{i-1}\left(\bar{z}^{i-1} - \bar{z}^i\right)^2}{w^i} \qquad (7)$$

$$v_{wg}^i = \frac{\sum w^{i-1} v_g^{i-1}}{w^i} \qquad (8)$$

$$v_g^i = v_{bg}^i + v_{wg}^i \qquad (9)$$

$$v_m^i = \frac{1}{w^i} \qquad (10)$$

$$n^i = \sum n^{i-1} \qquad (11)$$

$$n_{eff}^i = \frac{\left(w^i\right)^2}{w_{sq}^i} \qquad (12)$$

$$mv^i = \frac{n^i}{w^i} \qquad (13)$$

$$v^i = \begin{cases} v_m^i & if\ \dfrac{v_g^i}{mv^i} < \chi_{crit}^2 \\ v_g^i & if\ \dfrac{v_g^i}{mv^i} \geq \chi_{crit}^2 \end{cases} \qquad (14)$$

At each step the weights and squared weights are summed (4), (5) and the variance-weighted mean is computed (6). The variance for the group of data points $v_g$ is the sum (9) of the between-group variance $v_{bg}$ (7) and the within-group variance $v_{wg}$ (8). Between-group variance is the variance due to differences between group means, and the within-group variance is due to variances between values within the group, just as in an ANOVA. The variance of the mean for the group $v_m$ is equivalent to the inverse of the aggregated weight (10). The effective number of cells $n_{eff}$ is derived from the weights (12); it is equal to the number of cells $n$ when the weights are all equal but is less than $n$ when the weights are unequal. The mean noise variance in the group $mv$ is derived from the number of cells and the aggregated weight (13). The final step (14) compares the group variance with the mean noise variance and uses a statistical test to decide whether the group variance is small enough that the values in the group can be considered to be equal to the mean value, in which case it takes the variance of the mean as the variance at that resolution; otherwise it takes the group variance. The critical value $\chi_{crit}^2$ is computed with degrees of freedom equal to one less than the effective number of values $n_{eff}$.

This yields a nested series of means and variances at progressively coarser resolutions that can then be combined in the reverse sequence. This process is initialized with:

$$z_s^{i_{\max}} = \bar{z}^{i_{\max}},\ v_s^{i_{\max}} = v^{i_{\max}} \tag{15}$$

Then for each step $i$ from $i_{\max}$ down to 1:

$$\begin{aligned} z_s^{i,i-1} &= refine(z_s^i) \\ v_s^{i,i-1} &= refine(v_s^i) \end{aligned} \tag{16}$$

$$v_s^{i-1} = \left( \frac{1}{v^{i-1}} + \frac{1}{v_s^{i,i-1}} \right)^{-1} \tag{17}$$

$$z_s^{i-1} = \left( \frac{z^{i-1}}{v^{i-1}} + \frac{z_s^{i,i-1}}{v_s^{i-1}} \right) v_s^{i-1} \tag{18}$$

The smoothed coarse-scale elevations $z_s$ and variances $v_s$ are first refined to the next finer resolution (16), the variance calculated by aggregation at that resolution and the smoothed variance from the coarser resolution are combined (17) to produce the smoothed variance at the finer resolution, then the smoothed elevation is obtained by a weighted sum (18) of the elevation aggregated to that resolution and the smoothed elevation from the coarser resolution. The final result is $z_s^0$, the smoothed DEM, and $v_s^0$, the estimated variance.

Each step in the algorithm corresponds to a relatively simple raster calculation that can be implemented in a GIS. Using ArcInfo GRID, the sums in the first phase can be calculated using the AGGREGATE function over 3×3 cell groups and the refinements in the second phase can be calculated using a FOCALMEAN function, after suitably setting extents and cell sizes. There are some minor artifacts in the results that could be reduced using a more sophisticated refinement step.

The method assumes normally distributed and spatially uncorrelated noise; the degree to which divergence from those ideals affects the quality of the smoothing has not been investigated.

The statistical test (14) is probably the key distinguishing feature of this algorithm. It expresses the assumption that, where the grouped variance is low enough, the measured elevations in a local area should be considered to be randomly perturbed measurements of a single true elevation i.e. that the land surface is flat. The variance for that group (for the purpose of combining values in the refinement phase) is then the variance of the calculated mean, $v_m$, which is much smaller than the variance of the measurements. This low variance ensures that the mean dominates over the samples since $v_s^{i-1} << v^{i-1}$ in (17).

## ESTIMATING NOISE FROM A DEM

To apply this adaptive smoothing algorithm to DEM data an estimate of variance is required for each point. The method to estimate noise described here was developed for use with 1″ SRTM data (after destriping, filling voids and removing vegetation offsets, Gallant et al [5]) and was to some extent tuned to the characteristics of that DEM particularly the spatially correlated nature of the noise. Other data sources might require a different method; in some cases an estimate of the noise might be provided by the DEM production process.

For each cell a mean value is calculated over an annulus from 3 to 5 cells in radius; the annulus means that values near the target cell are excluded from the mean value. The difference between the target cell and the mean value is calculated, and the standard deviation of that difference over a 5 cell window is then derived. This provides information about the size of variation of elevations from the mean elevations a moderate distance away – the idea is that this variation should be mostly noise since modest topographic variation will produce spatially coherent differences from the mean which will contribute little to the standard deviation of differences.

This initial noise magnitude estimate is still quite erratic, so it is smoothed by two steps of median filtering, the first by aggregation over a 5×5 rectangle and the second over a circular window with a radius of 5 cells on this coarsened grid. The resulting grid is then refined back to the DEM resolution using bilinear resampling. The estimate corresponds to noise standard deviation.

Fig. 1 shows the results of this analysis on the 1 second SRTM DEM over a part of Western Australia where noise levels are highly variable.

Note that this method effectively distinguishes between noise and topography in low relief terrain with long slopes but is unable to make that distinction in higher relief areas or where there is an abrupt feature in otherwise low relief terrain. The strategy chosen to overcome this problem was to progressively reduce the estimated noise level as the standard deviation of elevation increased above 5 m.

## ADAPTIVE SMOOTHING OF SRTM

Fig. 2 shows shaded relief and Fig. 3 shows slope calculated from the 1 second SRTM DEM before and after application of the adaptive smoothing method using the noise estimate from

Figure 1. The shaded relief image highlights the smoothness of the low relief areas after adaptive smoothing. The low slopes of around 1–2% that dominate in this landscape are overwhelmed by the noise before smoothing. After smoothing the topographic slopes are clearly apparent. The steeper slopes in the south-eastern corner of the image are largely unaffected by the smoothing.

## DISCUSSION

This relatively simple adaptive smoothing algorithm effectively treats spatially varying noise in DEMs derived from dense remotely-sensed measurements. The method has also successfully been applied to lidar DEMs using a constant noise standard deviation of 0.2 m.

The smoothing algorithm also replaces areas of nodata with smoothed data from surrounding areas, due to the initialization with 0 weights in (3), which can be used as a simple method for filling voids.

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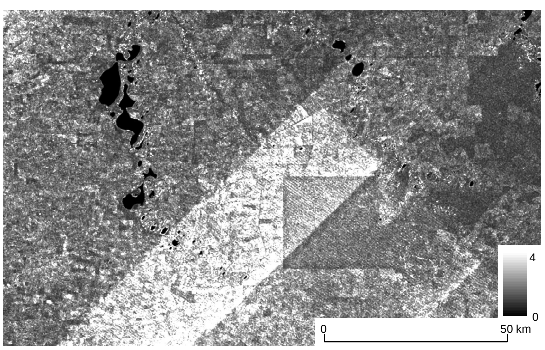


Figure 1. Noise standard deviation (m) estimated from SRTM data in Western Australia, 119.0E 33.6S. The square is an area of trees with higher radar reflectivity and hence lower noise than the surrounding cleared land

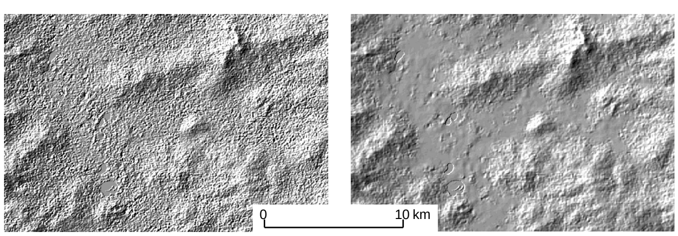


Figure 2. Shaded relief of a sub-section of the area in Figures 1 and 3 from SRTM data before (left) and after (right) adaptive smoothing.

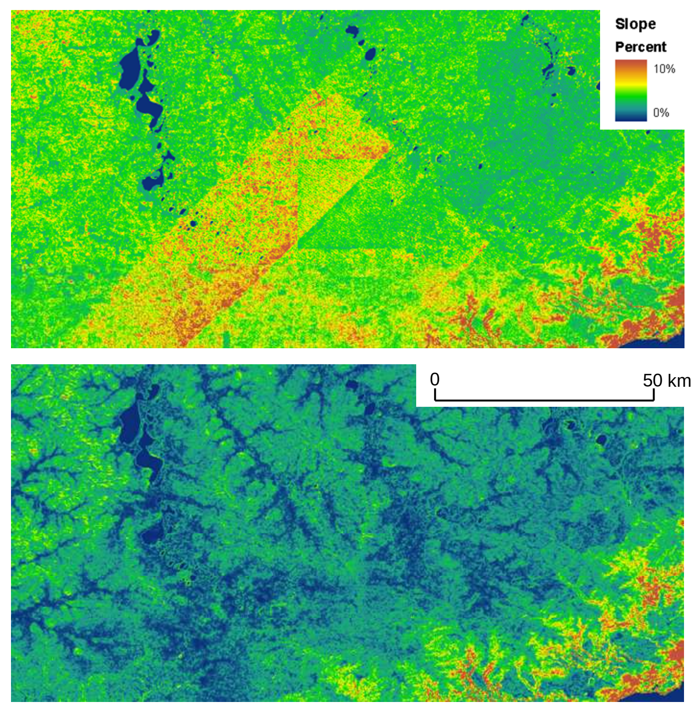


Figure 3. Slope calculated from 1” SRTM data before (top) and after (bottom) adaptive smoothing, using noise standard deviation of Figure 1.