# A Parameterisation Attempt of Scoria Cones of the San Francisco Volcanic Field (Arizona, USA) by Conical Fitting 

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## 1. Introduction

The availability of the Shuttle Radar Topography Mission (SRTM) Digital Terrain Model (DTMs) paved the way of new approaches in volcanic geomorphology. Because of their relatively well definable geometric form, the gradually degrading volcanic edifices shaped by the surface processes are suitable targets for morphometric studies. After the cessation of the volcanic activity the relatively steep slopes are gradually changing together with the height of the form. This process can be studied by the categorisation of the volcanic cones and parameter estimation on the different group members. During the degradation of volcanic edifices the characteristic properties of the cones change, so the geometric modelling of their surface may result in parameters that are meaningful in the understanding of these processes.

Our study area, the San Francisco Volcanic Field (SFVF), is a ca. $4500 \mathrm{~km}^{2}$-large volcanic region situated around the San Francisco stratovolcano in Arizona near Flagstaff (USA; Fig. 1).

The SFVF hosts some 600 scoria and lava domes, numerous lava flows with extensive volcanic ash deposits. Because of the wide range in size and age, as well as contrasting degradation of these volcanic features, several authors have analysed them in order to derive general rules of their lowering. Morphometric parameters were determined that were expected to be suitable to fulfil this requirement. In his pioneering work, Wood (1980a,b) considered 40 scoria cones, while almost two decades later Hooper and Sheridan (1998) included 237 features in their study. Their manual morphometric analyses were based on topographic maps that are time consuming, therefore their limited scope can now be extended with the availability of digital data.


Figure 1. An overview shaded relief map of the San Francisco Volcanic Field area.

## 2. Conceptual Considerations

The classic approach of the aforementioned authors focus on some parameters that can be estimated manually relatively easily from the topographic data. The height of the cone $\left(\mathrm{H}_{\mathrm{co}}\right)$, the width of the cone $\left(\mathrm{W}_{\mathrm{co}}\right)$, the width of the crater ( $\mathrm{W}_{\mathrm{cr}}$, if applicable) are the most important ones that were used in these seminal papers.

The application of digital data allows somewhat more complex approach: the fitting of the 3 -dimensional conical shape is also possible. However, the larger number of parameters increase the complexity of finding the best fitting model. In the case of five parameters, each point of the resulting five dimensional parameter space is a potentially fitting cone, while most of these models represented by the appropriate parameter values are far from being close to the shape to be modelled. On the other hand, the algorithms that are used to find the best fit models, tend to get stranded at local minima, failing to reveal the optimal solution.

An applicable general technique to help the algorithms to succeed is to figure out some parameters independently, that are later feed in the 3D processing. In our case the 2D profiles and planar (map view) contours were used for this purpose.

In order to facilitate further the minimization, the number of parameters has been reduced via the simplification of the model (e.g., the with the manual removal of the crater area) and replacement of the aforementioned classic parameters with simplistic ones.

## 3. Data Processing

In the initial phase of our project more than 300 cones were analysed using the classic approach (height of the cone, width of the cone and crater, etc.). Additionally the slope histogram were analysed in order to classify the cones into different evolutionary categories.

These analyses led to the selection of a few volcanoes, that entered in the next processing phase. We selected scoria cones according to their morphology: the basic requirement was to have a conical shape. We analysed the groups of cones with closed vent, cones with open vent and cones without vent. Initially a five-parameter 3D model
was conceptualised and the optimization is carried out using the simplex method (Nelder and Mead 1965):

$$
\begin{equation*}
Q_{\text {cone }}=\sum_{i=0}^{N}\left[\left(z_{0}-\frac{H}{R} \sqrt{\left(x_{i}-x_{0}\right)^{2}+\left(y_{j}-y_{0}\right)^{2}}+H\right)^{T}-h_{i j}{ }^{M}\right]^{2} \tag{1}
\end{equation*}
$$

where $Q_{\text {cone }}$ : value of the residual $N$ : number of the pixels
$z_{0}$ : elevation of the base plate level,
$R$ : radius of the base of the cone,
$x_{0}, y_{0}$ : coordinates of the centre,
$x_{i}, y_{j}$ : coordinates of the pixels,
$h_{i j}{ }^{M}$ : elevation data of the DTM.
$H$ : height of the cone,


Figure 2. Parameters of the fitted upright cone.
Because of the aforementioned complexity of the parameter estimation we reduced the number of the parameters to four (Fig. 2) using some simple geometrical considerations:

$$
\begin{equation*}
Q_{\text {cone }}=\sum_{i=0}^{N}\left[\left(c+a \sqrt{\left(x_{i}-x_{0}\right)^{2}+\left(y_{j}-y_{0}\right)^{2}}\right)^{T}-h_{i j}{ }^{M}\right]^{2} \tag{2}
\end{equation*}
$$

where $c=z_{0}+H, a=-\frac{H}{R}$, consequently $\alpha=-\arctan a$
and $\alpha$ is the elevation angle of cone (i.e., the slope angle of the volcanic edifice).
In order to achieve a better fit, some filtering of the DTM based on the slope distribution was carried out: we removed those points that were considered to be off from the sensu stricto volcanic cone (typically the products of the erosional processes at the hillfoot). Likewise we removed the crater area as well to avoid the bias introduced by this part of the volcanic feature.

## 4. Results

Most of the processed cones give a reasonably good fit, the residuals usually remain below $+/-30 \mathrm{~m}$. Residual maps (difference of the original DTM and the fitted cone) shows more or less the local roughness of the surface. An example for the residuals, the results for the SP Crater (Arizona, USA) is shown in Fig. 3.

Slope angles (values of $\alpha$ ) were calculated using the parameters of the fitted cones. These values were correlated with the age group of the volcanoes. In accordance with the expectations the slopes angles are ranging between $6^{\circ}$ and $30^{\circ}$, however most of the values are scattering in the lower angle ranges. Only those scoria cones of the Late Pleistocene and the Holocene give slopes above $24^{\circ}$ which are known to be the most recent edifices in this field while many of the cones are characterised by typical angles of $10^{\circ}, 12^{\circ}, 14^{\circ}$ and $19-20^{\circ}$ (Fig. 4). The results are in good congruence with the morphological and age groups and their evolution, however the quantization error in the slope angle histograms should also be considered in the case of underpopulated


Figure 3. The residual map overlain on the DTM of the original volcanic cone of SP Crater, Arizona. Note the residual pattern (see also in map view as inset in the bottom right) that implies a somewhat elliptical shape of the volcanic form). In the crater area the original elevation contours are drawn (all elevation values are in m ).

The results show that for a successful fit, only the most regular volcanoes are suitable. If the cone to be processed is well described by the 2D fit results in the preprocessing steps (this defines the initial simplex), in most of the cases a very good fit is
achievable. In this case, the residuals often remain below $+/-20 \mathrm{~m}$, while the standard deviation of the residuals is ca. 10 m . In comparison with the accuracy of the SRTM DTM this is very promising.

If the volcano is less regular, or the scoria cone is situated on a larger stratovolcano, there is necessary a break in the slope angle at the lower elevation parts, the fit is not so successful, giving residuals reaching 50 m . The parameters sometimes are tending to give a more general, average solution if there is some irregularity left in the data.


Figure 4. The elevation angle/slope angle histogram of the 50 processed SFVF scoria cones using $1^{\circ}$-wide bins. The highlighted bars indicate the angles where some tendency to clustering can be observed.

## 5. Conclusions

In the San Francisco Volcanic Field a number of scoria cones can be modelled by 3D cone fitting with acceptable accuracy. These cones are the youngest ones, characterised by regular shapes, but may also have craters that is not considered by our processing. However, even in these cases sometimes the parameter estimation method cannot find the optimal solution. Therefore the preprocessing step that preselects some parameters, may turn to be crucial in achieving a good fit. A typical good fit is characterised by maximal residual of $+/-20 \mathrm{~m}$, while the standard deviation of the residuals is in the order of magnitude of the accuracy of the elevation data. Elongated forms may also be modelled, but they result in larger residuals.

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