CHAPTER 1

Geomorphometry: A Brief Guide

R.J. Pike, I.S. Evans and **T. Hengl**

basic definitions · the land surface · land-surface parameters and objects · digital elevation models (DEMs) · basic principles of geomorphometry from a GIS perspective · inputs/outputs, data structures & algorithms · history of geomorphometry · geomorphometry today · data set used in this book

1. WHAT IS GEOMORPHOMETRY?

Geomorphometry is *the science of quantitative land-surface analysis* (Pike, 1995, 2000a; Rasemann et al., 2004). It is a modern, analytical-cartographic approach to representing bare-earth topography by the computer manipulation of terrain height (Tobler, 1976, 2000). Geomorphometry is an interdisciplinary field that has evolved from mathematics, the Earth sciences, and — most recently — computer science (Figure 1). Although geomorphometry[1] has been regarded as an activity within more established fields, ranging from geography and geomorphology to soil science and military engineering, it is no longer just a collection of numerical techniques but a discipline in its own right (Pike, 1995).

It is well to keep in mind the two overarching modes of geomorphometric analysis first distinguished by Evans (1972): *specific*, addressing discrete surface features (i.e. *landforms*), and *general*, treating the continuous land surface. The morphometry of landforms *per se*, by or without the use of digital data, is more correctly considered part of *quantitative geomorphology* (Thorn, 1988; Scheidegger, 1991; Leopold et al., 1995; Rhoads and Thorn, 1996). Geomorphometry in this book is primarily the computer characterisation and analysis of continuous topography. A fine-scale counterpart of geomorphometry in manufacturing is *industrial surface metrology* (Thomas, 1999; Pike, 2000b).

The ground beneath our feet is universally understood to be the interface between soil or bare rock and the atmosphere. Just what to call this surface and its science of measurement, however, is less obvious. Numerical representation of the

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[1] The term, distinguished from morphometry in other sciences (e.g. biology), dates back at least to Neuenschwander (1944) and Tricart (1947).

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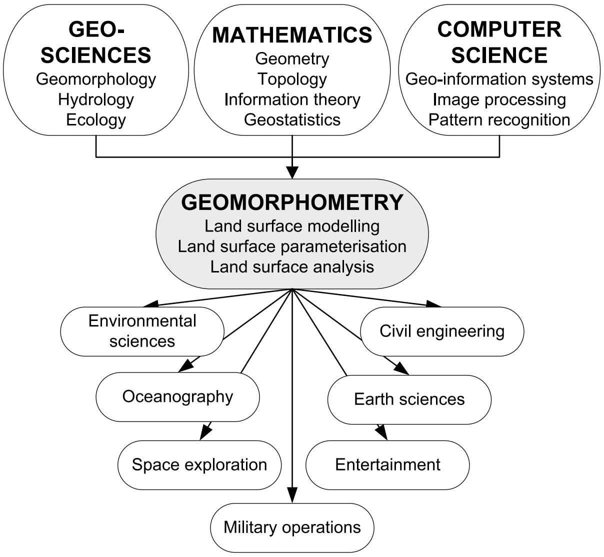

FIGURE 1 Geomorphometry and its relation to source and end-user disciplines. Modified after Pike (1995).

land surface is known variously as *terrain modelling* (Li et al., 2005), *terrain analysis* (Wilson and Gallant, 2000), or the *science of topography* (Mark and Smith, 2004).[2] Quantitative descriptors, or measures, of land-surface form have been referred to as *topographic attributes* or *properties* (Wilson and Gallant, 2000), *land-form parameters* (Speight, 1968), *morphometric variables* (Shary et al., 2002), *terrain information* (Martinoni, 2002), *terrain attributes* (Pennock, 2003), and *geomorphometric attributes* (Schmidt and Dikau, 1999).

> REMARK 1. *Geomorphometry is the science of topographic quantification; its operational focus is the extraction of land-surface parameters and objects from digital elevation models (DEMs).*

Despite widespread usage, as a technical term *terrain* is imprecise. *Terrain* means different things to different specialists; it is associated not only with land form, hydrographic features, soil, vegetation, and geology but also (like *topography*) with the socio-economic aspects of an area (Li et al., 2005). *Terrain*[3] also can signify an area of ground, a region... unrelated to *shape* of the land surface. The much used *terrain analysis* (Moore et al., 1991a; Wilson and Gallant, 2000) is confusing (unless preceded by *quantitative*), because it has long denoted qualitative (manual) stereoscopic photo- or image-interpretation (Way, 1973). Nor does the more precise *digital terrain modelling* (Weibel and Heller, 1991) escape ambiguity, as *terrain modelling* can infer measurement or display of surface heights, unspecified quantification of topography, or any digital processing of Earth-surface features.

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[2] The most frequent equivalents of *geomorphometry* in Google's online database appear to be *surface* or *terrain modelling, terrain analysis* and *digital terrain modelling* (Pike, 2002).

[3] *Terrain* is from the Latin *terrenum*, which might be translated as "*of the earth*".

Additionally, in many countries (e.g. France, Spain, Russia, Slovakia) *relief*[4] is synonymous with morphology of the land surface (King et al., 1999). This usage is less evident in Anglophone regions (e.g. Great Britain, North America), where *relief*, usually prefixed by *relative* or *local*, has come to denote the difference between maximal and minimal elevation within an area (Partsch, 1911; Smith, 1953; Evans, 1979), "*low*" and "*high*" relief indicating small and large elevation contrasts respectively.[5]

To minimise confusion, the authors of this book have agreed to consistently use *geomorphometry* to denote the scientific discipline and *land surface*[6] to indicate the principal object of study. Digital representation of the land surface thus will be referred to as a *digital land surface model* (DLSM), a specific type of *digital surface model* (DSM) that is more or less equivalent to the widely-accepted term *digital elevation model*[7] (DEM).

An area of interest may have several DSMs, for example, surface models showing slope gradient or other height derivative, the tree canopy, buildings, or a geological substrate. DSMs from laser altimetry (LiDAR, light detection and ranging) data can show more than one *return surface* depending on how deep the rays penetrate. Multiple DLSMs are usually less common but can include DEMs from different sources or gridded at different resolutions, as well as elevation arrays structured differently from square-grid DEMs (Wilson and Gallant, 2000). Objects of the built environment are of course not part of the land surface and must be removed to create a true bare-earth DLSM.

Digital elevation model (DEM) has become the favoured term for the data most commonly input to geomorphometry, ever since the U.S. Geological Survey (USGS) first began distribution of 3-arc-second DEMs in 1974 (Allder et al., 1982). Even *elevation* is not unique as it can also mean surface uplift (e.g. the Himalayas have an *elevation* of 5 mm/year). However, the alternative terms are less satisfactory: *height* is relative to a nearby low point, and *altitude* commonly refers to vertical distance between sea level and an aircraft, satellite, or spacecraft. Thus *digital height model* and *altitude matrix* (Evans, 1972) are avoided here.

REMARK 2. *The usual input to geomorphometric analysis is a square-grid representation of the land surface: a digital elevation (or land surface) model (DEM or DLSM).*

In this book, DEM refers to a gridded set of points in Cartesian space attributed with elevation values that approximate Earth's ground surface (e.g. Figure 5, below). Thus, contour data or other types of sampled elevations, such as a triangular array, are not DEMs as the term is used here. "DEM" implies that elevation is available continuously at each grid location, at a given resolution. See Chapter 2 for a detailed treatment of topography and elevation models.

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[4] fren. *Topographie*, germ. *Relief*, russ. рельеф, span. *Relieve*.

[5] This quantity is also known as *reliefenergie* (Gutersohn, 1932), particularly in Germany and Japan.

[6] fren. *Surface terrestre*, germ. *Gelände*, russ. земная поверхность, span. *Topografía*. A term that became widely known through the morphometric work of Hammond (1964).

[7] fren. *Modèle numèrique de terrain*, germ. *Digitales Gelände Model*, russ. цифровая модель рельефа, span. *Modelo de elevación digital*.

Finally, we define *parameter* and *object*, the two DEM-derived entities fundamental to modern geomorphometry (see, e.g., Mark and Smith, 2004). A *land-surface parameter*[8] is a descriptive measure of surface form (e.g. slope, aspect, wetness index); it is arrayed in a continuous field of values, usually as a raster image or map, for the same referent area as its source DEM. A *land-surface object*[9] is a discrete spatial feature (e.g. watershed line, cirque, alluvial fan, drainage network), best represented on a vector map consisting of points, lines, and/or polygons extracted from the square-grid DEM.

It is also important to distinguish parameters *per se*, which describe the land surface at a point or local sample area, from quantitative *attributes* that describe objects. For example, slope gradient at a given point refers only to its x, y location, whereas the volume of, say, a doline (limestone sink) applies to the entire area occupied by that surface depression; slope is a land-surface parameter, while depression volume over an area is an attribute of a land-surface object. Each of these quantities can be obtained from a DEM by a series of mathematical operations, or *morphometric algorithms*.

2. THE BASIC PRINCIPLES OF GEOMORPHOMETRY

2.1 Inputs and outputs

The fundamental operation in geomorphometry is *extraction of parameters and objects from DEMs* (Figure 2). DEMs, i.e. digital land-surface models, are the primary input to morphometric analysis. In GIS (geographic information system) terms, a DEM is simply a raster or a vector map showing the height of the land surface above mean sea level or some other referent horizon (see further Section 2 in Chapter 2).

Geomorphometry commonly is implemented in five steps (Figure 2):

1. *Sampling the land surface* (height measurements).
2. *Generating a surface model from the sampled heights*.
3. *Correcting errors and artefacts in the surface model*.
4. *Deriving land-surface parameters and objects*.
5. *Applications of the resulting parameters and objects*.

Land-surface parameters and objects can be grouped according to various criteria. Parameters commonly are distinguished as primary or secondary, depending on whether they derive directly from a DEM or additional processing steps/inputs are required (Wilson and Gallant, 2000). In this book, we will follow a somewhat different classification that reflects the purpose and type of analysis. Three main groups of land-surface parameters and objects are identified:

- *Basic morphometric parameters and objects* (see Chapter 6);
- *Parameters and objects specific to hydrology* (see Chapter 7);
- *Parameters and objects specific to climate and meteorology* (see Chapter 8);

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[8] fren. *Paramètre de la surface terrestre*, germ. *Reliefparameter*, russ. характеристика рельефа, span. *Variable del terreno*.

[9] fren. *Object de la surface terrestre*, germ. *Reliefobjeckt*, russ. объект земной поверхности, span. *Elemento del terreno*.

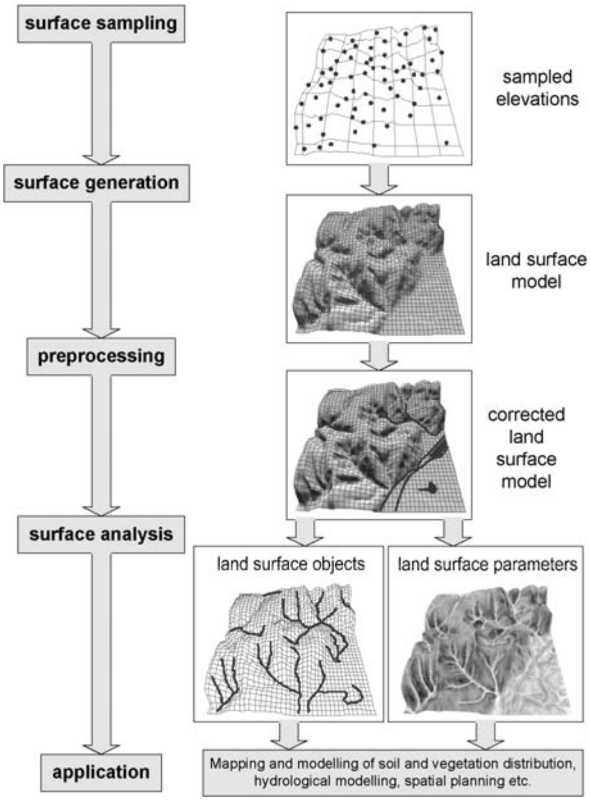

FIGURE 2 The operational focus of geomorphometry is extraction of land-surface parameters and objects from DEMs.

Basic parameters and objects describe local morphology of the land surface (e.g. slope gradient, aspect and curvature). Hydrological or flow-accumulation parameters and objects reflect potential movement of material over the land surface (e.g. indices of erosion or mass movement). The third group of parameters and objects is often calculated by adjusting climatic or meteorological quantities to the influence of surface relief.

A special group of land-surface objects — geomorphological units, *land elements* and *landforms* — receives its own chapter (Chapter 9). A landform is a discrete morphologic feature — such as a watershed, sand dune, or drumlin — that is a functionally interrelated part of the land surface formed by a specific geomorphological process or group of processes. Each landform may be composed of several landform elements, smaller divisions of the land surface that have relatively constant morphometric properties.

REMARK 3. *A landform element is a division of the land surface, at a given scale or spatial resolution, bounded by topographic discontinuities and having (relatively) uniform morphometry.*

Recognition of landforms and less exactly defined tracts, commonly referred to as *land-surface types*, from the analysis of DEMs is increasingly important. Many areas of the Earth's surface are homogeneous overall or structured in a distinctive way at a particular scale (e.g. a dune field) and need to be so delineated (Iwahashi and Pike, 2007). In the special case of landforms extracted as "*memberships*" by a fuzzy classification algorithm, such forms can be considered to "*partake*" of a particular land-surface object — instead of directly mapping, say, a stream channel, we can obtain a "*membership value*"[10] to that landform.

2.2 The raster data structure

Many land-surface representations, such as the background topography seen in video games and animated films, are modelled by mass-produced surface heights arrayed in some variant of the surface-specific *triangulated irregular network* (TIN) model (Blow, 2000; Hormann, 1969; see Chapter 2, Section 2.1). Most geomorphometric applications, however, use the square-grid DEM model. To be able to apply the techniques of geomorphometry effectively, it is essential to be familiar with the concept of a raster GIS and its unique properties.

Although the raster structure has a number of disadvantages, including a rectangular data array regardless of the morphology of the study area, large data-storage requirements, and under- and over-sampling of different parts of a diverse study area, it will remain the most popular format for spatial modelling in the foreseeable future. This structure is especially advantageous to geomorphometry because most of its technical properties are controlled automatically by a single measure: spatial resolution, *grid size* or *cell size*,[11] expressed as a constant x, y spacing (usually in metres) (Hengl, 2006).

In addition to grid resolution, we also need to know the coordinates of at least one grid intersection (usually marking the lower left-hand corner of the entire DEM array) and the number of rows and columns, whereupon we should be able to define the entire map (Figure 3). This of course assumes that the map is projected into an *orthogonal system* where all grid nodes are of exactly equal size and oriented toward cartographic North.

Accordingly, the small 6×6-pixel DEM in Figure 5 (see below) can also be coded in an ASCII file as an array of heights:

```
ncols 6
nrows 6
xllcorner 0
yllcorner 0
cellsize 10.00
nodata_value -32767
10 16 23 16 9 6
14 11 18 11 18 19
19 15 13 21 23 25
20 20 19 14 38 45
24 20 20 28 18 49
23 24 34 38 45 51
```

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[10] Such a value has been designated by the rather clumsy term *channelness*.

[11] *Cell size* is a more appropriate term than *grid size* because *grid size* can also imply size of the whole grid.

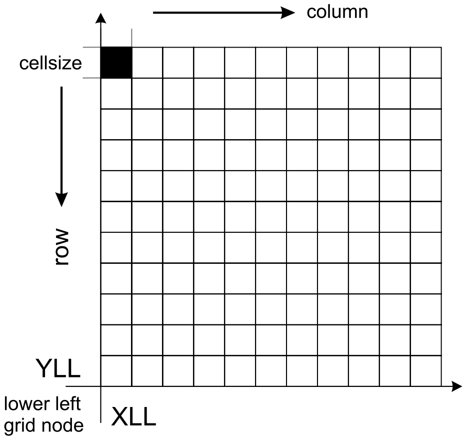

FIGURE 3 An orthogonal raster map can be defined by just five parameters: (a & b) number of rows and columns; (c & d) coordinates of the lower left corner and (e) cell size.

where `ncols` is number of columns, `nrows` is number of rows, `xllcorner` is the western edge of the map, `yllcorner` is the southern edge of the map, `cellsize` is grid resolution in metres, `nodata_value` is the arbitrary value used to mask out locations outside the area of interest and `10`, `16`, `23`, `16`, `9`, `6` are the elevation values in the (first) row. This is the standard format for ASCII grid files used by ESRI Inc. for its ArcInfo and ArcGIS software. It is necessary to define the initial point of the grid system correctly: there is a difference in x, y location of half the `cellsize`, depending on whether the first coordinate is at the lower left-hand corner of the lower left-hand grid cell (`llcorner`) or at the centre of that cell (`llcenter`).

> REMARK 4. *The principal advantage of a raster GIS over other spatial data structures is that a single measure — the cell or pixel size — automatically controls most technical properties.*

2.3 Geomorphometric algorithms

Performing morphometric operations within a raster GIS usually involves calculating intermediate quantities (over the same grid of interest) which are then used to compute the final output. Most morphometric algorithms work through the *neighbourhood operation* — a procedure that moves a small regular matrix of cells (variously termed a *sub-grid* or *filter window*) over the entire map from the upper left to the lower right corner and repeats a mathematical formula at each placement of this sampling grid.

Neighbouring pixels in a sampling window are commonly defined in relation to a central pixel, i.e. the location for which a parameter or an object membership is derived. In principle, there are several ways to designate neighbouring pixels, most commonly either by an identifier or by their position relative to the central

(*a*)

z_{NB1}	z_{NB2}	z_{NB3}
z_{NB4}	z_{NB5}	z_{NB6}
z_{NB7}	z_{NB8}	z_{NB9}

z_{NB1}	z_{NB2}	z_{NB3}	z_{NB4}	z_{NB5}
z_{NB6}	z_{NB7}	z_{NB8}	z_{NB9}	z_{NB10}
z_{NB11}	z_{NB12}	z_{NB13}	z_{NB14}	z_{NB15}
z_{NB16}	z_{NB17}	z_{NB18}	z_{NB19}	z_{NB20}
z_{NB21}	z_{NB22}	z_{NB23}	z_{NB24}	z_{NB25}

(*b*)

-1, 1	0, 1	1, 1
-1, 0	0, 0	1, 0
-1, -1	0, -1	1, -1

-2, 2	-1, 2	0, 2	1, 2	2, 2
-2, 1	-1, 1	0, 1	1, 1	2, 1
-2, 0	-1, 0	0, 0	1, 0	2, 0
-2, -1	-1, -1	0, -1	1, -1	2, -1
-2, -2	-1, -2	0, -2	1, -2	2, -2

FIGURE 4 The common designation of neighbours in 3×3 and 5×5 window environments: (a) by unique identifiers (as implemented in ILWIS GIS), (b) by row and column separation (in pixels) from the central pixel (as implemented in the ArcInfo GIS).

pixel (Figure 4). The latter (e.g. implemented by the `DOCELL` command in ArcInfo) is the more widely used because it can readily pinpoint almost any of the neighbouring cells anywhere on the map [Figure 4(b)].

Computing a DEM derivative can be simple repetition of a given formula over the area of interest. Consider a very small DEM of just 6×6 pixels. You could zoom into these values (elevations) and derive the desired parameter on a pocket calculator (Figure 5). For example, using a 3×3 sampling window, slope gradient at the central pixel can be derived as the average change in elevation. Three steps are required; first, the difference in relative elevation is calculated in x and y directions, whereupon slope gradient is obtained as the average of the two quadratics (Figure 5). By the Evans–Young method[12] (Pennock et al., 1987), slope gradient is calculated (see further Chapter 6):

$$G = \frac{z_{\mathrm{NB3}} + z_{\mathrm{NB6}} + z_{\mathrm{NB9}} - z_{\mathrm{NB1}} - z_{\mathrm{NB4}} - z_{\mathrm{NB7}}}{6 \cdot \Delta s}$$

$$H = \frac{z_{\mathrm{NB1}} + z_{\mathrm{NB2}} + z_{\mathrm{NB3}} - z_{\mathrm{NB7}} - z_{\mathrm{NB8}} - z_{\mathrm{NB9}}}{6 \cdot \Delta s}$$

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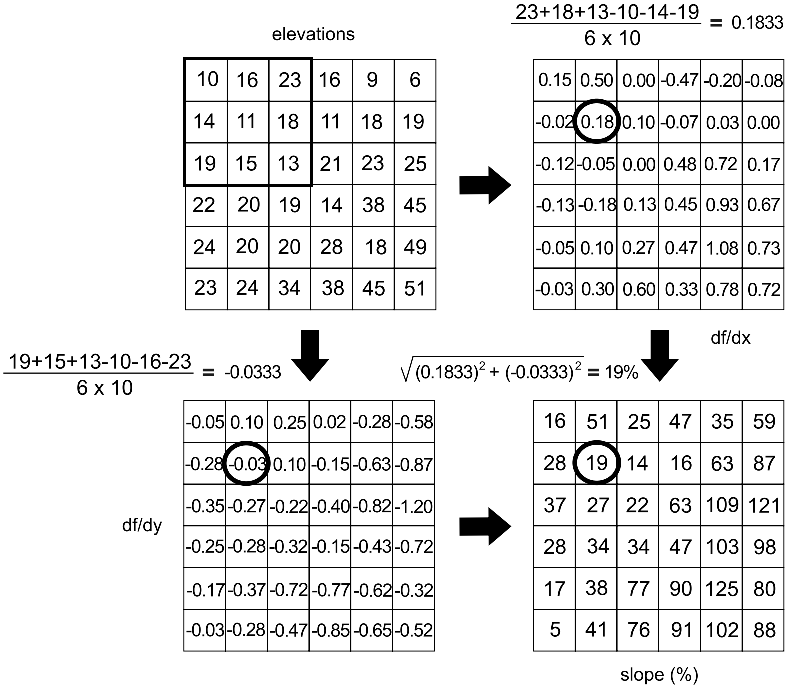
[12] Often, one land-surface parameter can be calculated by several different formulas or approaches; we caution that the results can differ substantially!



elevations

10	16	23	16	9	6
14	11	18	11	18	19
19	15	13	21	23	25
22	20	19	14	38	45
24	20	20	28	18	49
23	24	34	38	45	51

df/dx

0.15	0.50	0.00	-0.47	-0.20	-0.08
-0.02	0.18	0.10	-0.07	0.03	0.00
-0.12	-0.05	0.00	0.48	0.72	0.17
-0.13	-0.18	0.13	0.45	0.93	0.67
-0.05	0.10	0.27	0.47	1.08	0.73
-0.03	0.30	0.60	0.33	0.78	0.72

df/dy

-0.05	0.10	0.25	0.02	-0.28	-0.58
-0.28	-0.03	0.10	-0.15	-0.63	-0.87
-0.35	-0.27	-0.22	-0.40	-0.82	-1.20
-0.25	-0.28	-0.32	-0.15	-0.43	-0.72
-0.17	-0.37	-0.72	-0.77	-0.62	-0.32
-0.03	-0.28	-0.47	-0.85	-0.65	-0.52

slope (%)

16	51	25	47	35	59
28	19	14	16	63	87
37	27	22	63	109	121
28	34	34	47	103	98
17	38	77	90	125	80
5	41	76	91	102	88

FIGURE 5 Numerical example showing slope tangent (in %) extracted from a DEM using a 3×3 window.

where G is the first derivative in the x direction (df/dx), H is the first derivative in the y direction (df/dy), z_{NB5} is the (central) cell for which the final value of slope is desired, $z_{\mathrm{NB1,2,3,4,6,7,8,9}}$ are the eight neighbouring cells, and Δs is pixel size in metres (Figure 5). The slope gradient as a tangent is finally computed as:

$$\mathrm{SLOPE} = \sqrt{H^2 + G^2}$$

Note that the example in Figure 5 shows values of slope gradient for rows and columns at the edge of the map, although we did not actually have the necessary elevation values outside the map area. Normally, a neighbourhood operation is possible only at a grid location surrounded by its eight immediate neighbours. Because keeping to this practice loses the outermost rows and columns, the expedient solution illustrated in this example is to estimate missing neighbours by duplicating cells at the edges of the DEM and tolerating the (usually) modest error in the final result. By so doing, the output map retains exactly the same size as the input map.

REMARK 5. *Because most land-surface parameters vary with spatial scale, or can be calculated by different algorithms and sampling grids, no map computed from a DEM is definitive.*

Adjustments such as these differ among software packages, so that almost always some small differences will be found in outputs from exactly the same mathematical formulas. To avoid confusion, in referring to various types of general land-surface parameters and objects we will consistently specify (1) the algorithm (reference), (2) size of the sampling window and (3) the cell size. The example above would be slope (*land-surface parameter type*) calculated by the Evans–Young method (Pennock et al., 1987) (*variant*) in a 3×3 window environment (*sub-variant*) using a 10 m DEM (*cell size*). The rounding factor also can be important because some intermediate quantities require high precision (many decimal places), while others must never equal zero or take a negative value.

Finally, in Figure 5 we can see that the pixel with highest slope, 125%, is at location `row` = 5, `column` = 5 and the lowest slope, 5%, is at location `row` = 6, `column` = 1. Of course, in a GIS map the heights are rarely represented as numbers but rather by colour or greyscale legends.

3. THE HISTORY OF GEOMORPHOMETRY

Before exploring data, algorithms and applications in detail, it is well to step back and consider the evolution of geomorphometry, from the pioneering work of German geographers and French and English mathematicians to results from recent Space Shuttle and planetary-exploration missions. While its ultimate origins may be lost in antiquity, geomorphometry as we know it today began to evolve as a scientific field with the discoveries of Barnabé Brisson (1777–1828), Carl Gauss (1777–1855), Alexander von Humboldt (1769–1859), and others, reaching maturity only after development of the digital computer in the mid- to late-20th century.

REMARK 6. *Geomorphometry evolved from a mix of mathematics, computer processing, civil and military engineering, and the Earth sciences — especially geomorphology.*

The earliest geomorphometry was a minor sub-activity of exploration, natural philosophy, and physical geography — especially geomorphology; today it is inextricably linked with geoinformatics, various branches of engineering, and most of the Earth and environmental sciences (Figure 1). In the following sections we will briefly describe the approaches and concepts of pre-DEM morphometry as well as analytical methods applied to contemporary data. Additional background is available in Gutersohn (1932), Neuenschwander (1944), Zakrzewska (1963), Kugler (1964), Hormann (1969), Zavoianu (1985), Krcho (2001), and Pike (1995, 2002).

3.1 Hypsometry and planimetric form

Geomorphometry began with the systematic measurement of elevation above sea level, i.e. land surveying — almost certainly in ancient Egypt.[13] Height measurement by cast shadows is ascribed to the Greek philosopher Thales of Miletus

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[13] Land surveying that focuses on measurement of terrain height is often referred to as *hypsometry*, from the Greek *χυπςος* — height.

(ca. 624–546 B.C.). The concept of the elevation contour to describe topography dates to 1584 when the Dutch surveyor Pieter Bruinz drew lines of equal depth in the River Spaarne; but this was an unpublished manuscript (Imhof, 1982). In 1725 Marsigli published a map of depth contours in the Golfe du Lion, i.e. the open sea. In 1737 (published in 1752) Buache mapped the depth of the Canal de la Manche (English Channel), and in 1791 Dupain-Triel published a rather crude contour map of France (Robinson, 1982, pp. 87–101/210–215).

In 1774, British mathematician Charles Hutton was asked to summarise the height measurements made by Charles Mason,[14] an astronomer who wanted to estimate the mass of Earth. Hutton used a pen to connect points of the same height on the Scottish mountain Schiehallion, developing the *isohypse* (or isoline) concept. This has proved very effective in representing topography and is one of the most important innovations in the history of mapping by virtue of its convenience, exactness, and ease of perception (Robinson, 1982). DeLuc, Maskelyne, Roy, Wollaston, and von Humboldt were among many early investigators who used the barometer invented by Evangelista Torricelli (1608–1647) and developed by Blaise Pascal (1623–1662) to measure elevation; see also Cajori (1929) and de Dainville (1970).

With the spread of precise surveying in late 18th- and early 19th-century Europe, illustrations ranking mountain-top elevations and the lengths of rivers began to appear in atlases.[15] Mountain heights and groupings were studied qualitatively, often by military engineers (von Sonklar, 1873), as *orography*, their heights and derived parameters as *orometry* (Figure 6). Early 19th-century German geographers such as von Humboldt (recently cited in Pike, 2002, and Rasemann et al., 2004) compared summit heights in different ranges. Von Sonklar (1873), and earlier regional monographs, went further and considered the elevations of summits, ridges, passes and valleys as well as relative heights, gradients and volumes. *Orometry* — with emphasis on mean slope, mean elevation and volume, planimetric form, relative relief, and drainage density — became a favoured dissertation topic for scores of European geographers (Neuenschwander, 1944). The overarching charter of geomorphometry was nicely captured many years ago by the German geographer Alfred Hettner (1859–1941), when he wrote in a brief consideration and critique of 19th-century orometry: "*But it is more important to enquire whether we cannot express the entire character of a landscape numerically*" (Hettner, 1928, p. 160; republished in 1972).

Before the wider availability of contour maps in the mid-19th century,[16] most quantitative analyses of topography were of broad-scale linear features: rivers and coasts. The concavity of longitudinal river profiles, adequately determined from spot heights, came to be represented by exponential and parabolic equations (Chorley et al., 1964, §23). Carl Ritter (1779–1859) introduced indices of *Küstenentwicklungen* (*Coastal Development*) to distinguish intricate coastlines such as fjords from simpler ones such as long beaches. Some indices were more descriptive than

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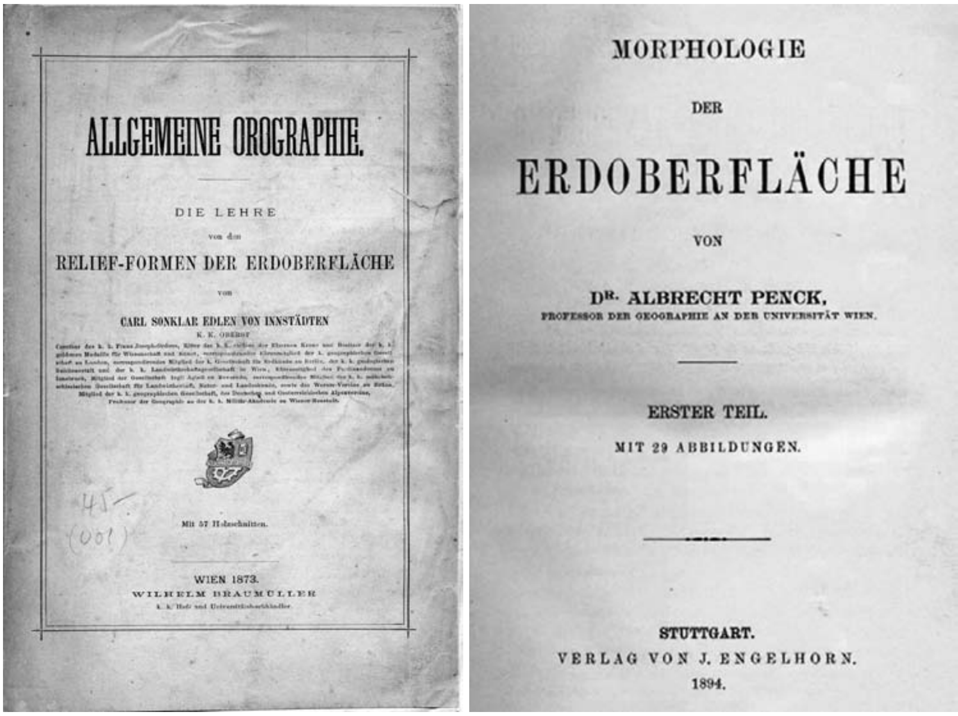
[14] This is the same Charles Mason who, with Jeremiah Dixon, surveyed the Mason–Dixon Line in the USA between 1763 and 1767.

[15] Tufte (1990, p. 77) reproduces just such a detailed 1864 diagram from J.H. Colton.

[16] Because early topographic maps represented relief by hachures, not contours, analysis of slope required detailed field survey and thus was rare.



ALLGEMEINE OROGRAPHIE.

DIE LEHRE
von den
RELIEF-FORMEN DER ERDOBERFLÄCHE
von
CARL SONKLAR EDLEN VON INNSTÄDTEN
K. K. OBERST

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Mit 57 Holzschnitten.

WIEN 1873.
WILHELM BRAUMÜLLER
k. k. Hof- und Universitätsbuchhändler.

MORPHOLOGIE
DER
ERDOBERFLÄCHE
VON
Dr. ALBRECHT PENCK,
PROFESSOR DER GEOGRAPHIE AN DER UNIVERSITÄT WIEN.

ERSTER TEIL.
MIT 29 ABBILDUNGEN.

STUTTGART.
VERLAG VON J. ENGELHORN.
1894.

FIGURE 6 Two landmarks of early geomorphometry from Germany and Austria, arguably the cradle of geomorphometry. The brief 19-page chapter on *orometrie* in von Sonklar's 1873 textbook (left) presented twelve quantitative measures of mountain morphology, which stimulated much publication on land-surface characterisation. One of the best summary treatments of early geomorphometry (including criticism of Sonklar!) was a much longer and wider-ranging chapter in Penck's 1894 textbook (right). Photos by R. Pike.

others; the ratio of an island's area to the square of its perimeter, for example, combined coastal sinuosity with compactness, whereas the ratio of its area to area of the smallest circumscribed circle was only an inverse measure of elongation, not *circularity* as claimed.

The impossibility of agreeing on a definitive length for a section of coastline eventually led to Richardson's (1961) establishment of a scaling relation between step length (i.e. measurement resolution) and estimated line length, and later the *fractal concepts* (Mandelbrot, 1967, 1977) of self-similarity and non-Euclidean form. As Mandelbrot's (1967) title implies, these widely applied scaling concepts were firmly rooted in coastal geomorphometry.[17]

Once contour maps were more available, relief analysis flourished. Measurement of highest and lowest points within a sample area (commonly a square or circle) quantified the vertical dimension as *relief* (*Reliefenergie* in German), which developed from the need to express relative height (Gutersohn, 1932). Partsch

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[17] Much further evidence could have been found in Volkov (1950), not cited by Mandelbrot (see also Maling, 1989, pages 277–303, and pages 66–83 citing the 1894 measurements of A. Penck on the Istrian coast).

(1911) used elevation range per 5×5 km square to produce what probably is the first quantitative map of local (relative) relief. Other definitions expressed relief for a hillslope (ridge crest to valley floor) or for a fluvial drainage basin: "catchment" or "watershed" relief (Sherman, 1932). Attempts to define relief as the separation between an upper *relief envelope* or summit surface and a valley or *streamline surface* (reviewed in Rasemann et al., 2004) were less successful because of scale variations. Working for the U.S. Army, W.F. Wood (1914–1971) quantified the dependency of relief upon area by statistical analysis of 213 samples measured on U.S. contour maps (Wood and Snell, 1957).

Geographers and later geomorphologists planimetered the areas enclosed by contours to generate plots of elevation versus area. Estimates for the entire globe by Murray (1888) were rough but sufficient to establish the bimodality of Earth's elevations, peaking near 0 and −4600 m, which posed numerous questions for geologists and geophysicists. This *hypsographic curve* could be cumulated and integrated for comparative studies of regions (de Martonne, 1941). The histograms of de Martonne (1941) are misleading, however, because he used two class intervals with the same linear vertical scale.

The dimensionless hypsometric integral, first applied to *landforms* (cirques) by Imamura (1937) and to regions by Péguy (1942), approaches zero where a few high points rise above a plain, and 1.0 where most surface heights cluster near the maximum. Although this device is useful morphologically and in geomorphology, hydrologic and other applications often require retention of landform dimensions. Strahler (1952) popularised an integral of the hypsometric curve, which later was proven identical to a simpler measure as well as the approximate reciprocal of elevation skewness[18] (Pike and Wilson, 1971). Péguy (1948) called further for a more conventional statistical approach and proposed the standard deviation of elevation as a measure of relief because of instability of the maximum. He asserted: "*Like all adult science, the geography of the second half of this century will be called to make more and more continuous appeal to mathematical methods*" (Péguy, 1948, p. 5).

Clarke (1966) critically reviewed hypsometry, clinometry and altimetric analysis, which had often been used in the search for old erosion (planation) surfaces over the prior 40 years. He showed that several types of clinographic curves, going back to the earliest examples by Sebastian Finsterwalder and Carl Peucker in 1890, can be misleading in their attempts to plot average slope gradient against elevation.

3.2 Drainage topology and slope frequency

In 1859, Alfred Cayley published "*On contour lines and slope lines*", which laid out the mathematical foundation of geomorphometry.[19] In this extraordinary paper, the land surface is considered in the gravitational field, and thus certain lines and points are more significant than others. Cayley defined *slope lines* as being always at right angles to contours. On a smooth, single-valued surface, all slope lines run from summits to pits (ultimately the ocean), except those joining summits (*ridge*

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[18] See further Figure 4 in Chapter 28.

[19] He was preceded by even earlier French mathematicians and geometers (Pike, 2002).

lines) and those joining pits (*course lines*). *Passes* are the lowest points on the former, and *pales* are the highest points on the latter. Each pass and pale is located at the intersection of a ridge line and a course line.

James Clerk Maxwell (1870) further noted that each territory defined by these special lines was part of both a *hill* whose lines of slope run down from the same summit, and a *dale* whose slope lines run down to the same pit. Hills are bounded by course lines, and dales by ridge lines. These pioneering *semantics* remained neglected until their rediscovery by Warntz (1966, 1975) and Mark (1979). They have since been again rediscovered by the engineering-metrology community (Scott, 2004).

Fluvial geomorphometry evolved from concepts of stream frequency (and its reciprocal, drainage density) and stream order, notably in the pioneering work of Ludwig Neumann and Heinrich Gravelius (Neuenschwander, 1944). The quantitative study of rivers and river networks initially was dominated by hydraulic engineers rather than geographers or geomorphologists, the work of Horton (1932, 1945) on network topology and related geometric attributes of drainage basins being especially influential. His revolutionary 1945 synthesis of hydrology and geomorphology rapidly evolved into the sub-field of drainage network analysis in the 1950s and 1960s (Shreve, 1974), which grew to such an extent that elaboration of stream-order topology overshadowed geometric analysis of the land surface.

Many geomorphological studies from the 1960s through the 1980s sought to relate hillslopes to streams (see later section) and in so doing exhaustively parameterised the shape and relief of individual drainage basins (Zavoianu, 1985; Gardiner, 1990). The *drainage basin* is Earth's dominant land-surface object and its analysis is, strictly speaking, a branch of specific geomorphometry. However, fluvial networks occupy so high a fraction of Earth's surface that the analysis of distributed drainage systems has come to dominate the more process-oriented implementations of general geomorphometry (Rodríguez-Iturbe and Rinaldo, 1997).

Statistical analysis of large samples of slopes began with Strahler's (1950) work in southern California, leading to the Columbia School of quantitative and dynamic fluvial geomorphology (Morisawa, 1985). Strahler measured maximum slope down a hillside profile (flow-line) and mean (overall) gradient, and related both to the gradient and topological order of the stream below. Tricart and Muslin (1951) advocated measuring large samples of 100 to 200 slope gradients from crest to foot on maps, in degrees rather than percentage; histograms for a homogeneous sample area tended to be symmetric and conspicuously peaked. Adapting a technique from structural geology, Chapman (1952) added a third dimension to slope analysis by treating planar surfaces as '*poles to the plane*'. He constructed radial plots of slope gradient against aspect (calculated from a gridded sample of points) to visually interpret asymmetry and lineation, an approach subsequently incorporated in the MicroDEM package (Guth et al., 1987).

The adoption of frequency distributions and statistical tests represented considerable progress and was promoted by Chorley (1957, 1966) for both drainage basins and individual slope segments. Tricart (1965) critically reviewed slope and fluvial morphometry, asserting that scale cannot be ignored if river profiles and channel incision are to be related to slope processes (Schumm, 1956). Yet despite such advances, the more dominant view among geologists and geographers in the early- to mid-1950s remained: "*mathematical analyses of topographic maps... are tedious, time-consuming, and do not always yield results commensurate with the amount of time required for their preparation*" (Thornbury, 1954, p. 529).[20]

Hormann (1969) brought a more distributed context to topographic analysis by devising a Triangulated Irregular Network (TIN), linking selected points on divides, drainage lines and breaks in slope to interrelate height, slope gradient, and aspect. Rather than individual data points, Hormann plotted averages over intervals, but also was able to consider valley length, depth, gradient, and direction. Criticised by one German colleague as excessively coarse and mechanistic, Hormann's TIN model was successfully developed in North America (Peucker and Douglas, 1975). Its surface-specific vector structure, complementary to the raster square-grid model, has since become a staple of both geomorphometry and GIS packages (Jones et al., 1990; Weibel and Brandli, 1995; Tucker et al., 2001).

Slopes had been profiled in the field (down lines of maximum gradient) in the 19th century (Tylor, 1875), but early geomorphometricians calculated slope from the contour spacing on maps[21] (as illustrated in Figure 7). As geomorphologists grew dissatisfied with the inadequacies of contour maps, field measurement of gradients and profiles became widespread in the 1950s. Slope profiling developed especially in Britain where many contours were interpolated yet photogrammetry was regarded as inadequate by the official mapping agency. Slope profiles were surveyed either in variable-length segments or with a fixed 1.52 m frame (Young, 1964, 1972; Pitty, 1969)[22]; still, a truly random sample of sinuous lines from a rough surface proved elusive. One motive for plotting frequency distributions of slope gradient was to discover characteristic slope angles, and upper and lower limiting angles relevant to slope processes (Young, 1972, pp. 163–167). Parsons (1988) reviewed further developments in slope profiling and slope evolution.

Local shape of the land surface is largely a function of curvature, or change of slope, a second derivative of elevation (Minár and Evans, 2008). Its importance in both profile and plan for hydrology and soils has long been recognised (Figure 7) and it forms the basis of a generic nine-fold (3×3) classification into elementary forms that are *convex*, *straight* or *concave* in plan, and in profile (Richter, 1962). This appealing taxonomy is useful, but precisely what constitutes a *straight* (i.e. planar) slope must be defined operationally; e.g. Dikau (1989) used a 600 m radius of curvature as the threshold of convexity and concavity (see further Figure 7 in Chapter 9).

The breaks and inflections of slope that delimit elementary forms or facets of the land-surface form the basis of morphographic mapping, a subset of geomorphological mapping which we shall not review in detail here (Kugler, 1964; Young, 1972; Barsch, 1990). Morphography is based on field mapping and air-

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[20] Even more severe was the criticism of Wooldridge (1958), who wrote disparagingly: "*At its worst this is hardly more than a ponderous sort of cant... If any best is to result from the movement, we have yet to see it'...*"

[21] Average slope could be estimated from the density of contour intersections with a grid (Wentworth, 1930).

[22] Equal spacing of profiles along a mid-slope line provided better coverage than starting from the slope crest or foot (Young, 1972, p. 145).

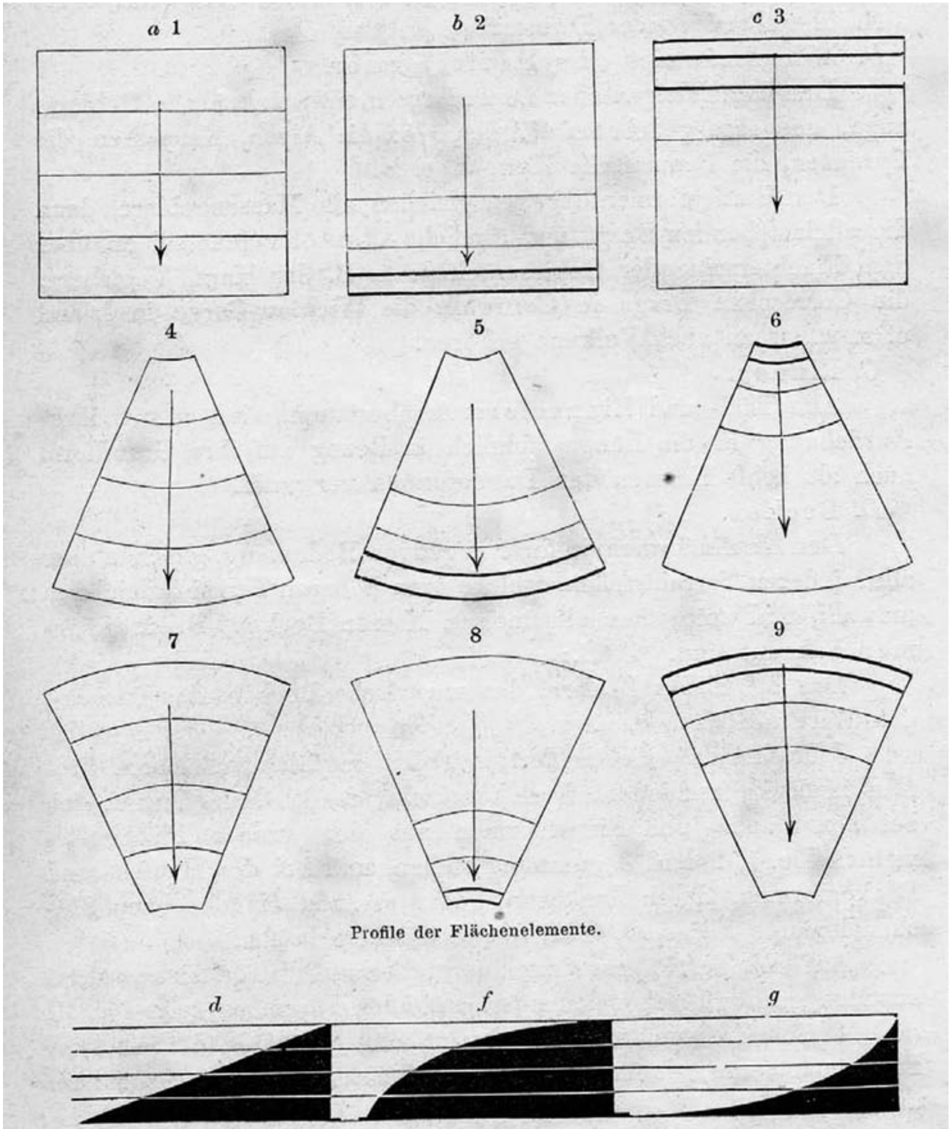

FIGURE 7 Illustration of the nine basic elements of surface form in the 1862 textbook on military geography by an Austrian army officer, long pre-dating 20th-century morphometry (see further Chapter 9). Photo by R. Pike.

photo interpretation, but a number of recent papers have attempted to automate the practice from DEMs, with varying success (see further Chapter 22).

3.3 Early DEMs and software tools

World War II innovation in technology set the stage for postwar advances in geomorphometry, many of which were inaccessible or poorly circulated due to defence-related sponsorship. Pike (1995) asserted that the field is unlikely to have developed as it did without the Cold War (1946–1991) and its space exploration offshoots[23] (Cloud, 2002). Some of the limited-distribution American reports from the 1950s and 1960s that stimulated general geomorphometry are listed by Zakrzewska (1963) and Pike (2002). Wood and Snell (1960), for example, manually measured six factors (in order of importance: average slope, grain, average elevation, slope direction changes, relative relief, and the elevation–relief ratio) from contour maps for 413 sample areas in central Europe, to delimit 25 land surface regions — a model for subsequent multivariate regionalisation by computer. Before the end of the decade W.F. Wood, M.A. Melton (1958), and others were beginning to tabulate topographic data on punched cards.

With emergence of the digital computer in the early- to mid-1950s, the progress of geomorphometry accelerated rapidly. The first input data were not DEMs but point elevations and topographic profiles. *Trend-surface analysis*, for example, numerically separates scattered map observations into two components, regional and local. The technique assumes that a spatial distribution can be modelled numerically as a continuous surface, usually by a polynomial expression, and that any observed spatial pattern is the sum of such a surface plus a local, *random*, term. Much used on subsurface data in petroleum exploration, by the 1960s it had attracted the attention of geomorphologists, notably to confirm planation surfaces or enhance local surface features (Krumbein, 1959; King, 1969). Trend-surface analysis commonly yields results as a square-grid array, but the polynomial fits to elevation data frequently oversimplified real-world variations in the topography.

The early numerical descriptions of topographic profiles were carried out by *spectral analysis*, a mathematical technique from signal processing and engineering that displayed the observations by spatial frequency (Bekker, 1969). First used to quantify the roughness of aircraft runways from surveyed micro-relief elevation profiles (Walls et al., 1954), elevation spectra were calculated from lunar surface measurements to support design of the Moon-landing program's Roving Vehicle (Jaeger and Schuring, 1966). To target lunar imaging missions, J.F. McCauley and colleagues at USGS had earlier (1963–64) computed slope gradient from topographic profiles generated through Earth-based photoclinometry ("*shape from shading*") of the Moon's surface (Bonner and Schmall, 1973). These data were also used to quantify the scale-dependency of slope gradient. Although single linear profiles capture apparent rather than true (maximum gradient) slopes[24] and do not deliver

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[23] For example, the U.S. Navy funded Strahler and E.H. Hammond, and later T.K. Peucker and David Mark (in Canada). Ian Evans' early work was supported by the U.S. Army and that of Pike by the Army and the National Aeronautics and Space Administration; the library of small DEMs (Tobler, 1968) that inspired both of us was funded by the Army.

[24] *Mean* apparent slope is correctable to its true value by multiplying by 1.5708 (i.e. $\pi/2$).

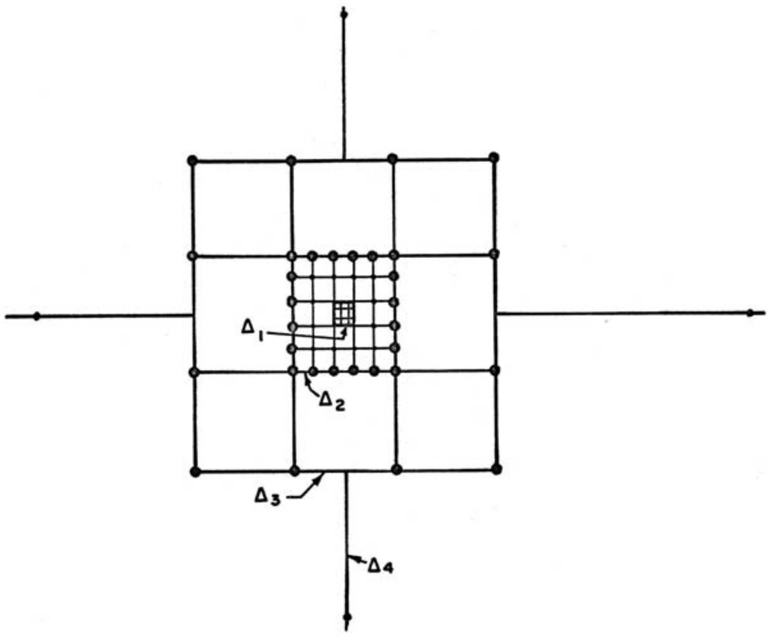

FIGURE 8 The earliest representation of a gridded x, y DEM designed to quantify variation in line-of-sight visibility with spatial scale. Grid spacings $\Delta_1 - \Delta_3$ of nested arrays (each 34×34 elevations) were 180, 800, and 9650 m. From unclassified 1959 American Association for the Advancement of Science symposium presentation by Arthur Stein.

the full 3-D character of a surface, spectral analysis continued to support morphometric objectives, such as delimiting morphologic regions of the seafloor (Fox and Hayes, 1985).

By the mid- to late-1950s, arrays of gridded elevations were being prepared by geophysicists for gravity correction, by civil engineers for highway location, and by the military in classified research on tactical combat doctrine. The DEM concept was first described openly by Miller and Laflamme (1958) at the Massachusetts Institute of Technology[25] but did not come into general use until the 1960s. Its potential and importance were clouded by limitations of the mainframe computers of the day. Although some DEMs were prepared from direct photogrammetry or field survey, most of them were laboriously interpolated by hand from existing contour maps[26] (e.g. Tobler, 1968). Semi-automated digitising of the entire United States at a grid resolution of about 63 m from 1:250,000-scale contour maps over 1963–1972 (Noma and Misulia, 1959; U.S. Army Map Service, 1963), later distributed by the USGS, marked a breakthrough in DEM availability. First and second surface derivatives (of gravity data) had aided in petroleum exploration; their calculation for

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[25] Cloud (2002) writes: "*Much of the primary development work was done by staff at the MIT Photogrammetric Laboratory, under contract to the Army/Air Force nexus*"; see also Figure 8.

[26] By 1964, W.F. Wood at Cornell Aeronautical Laboratory was creating DEMs to model line-of-sight calculations.

the land surface by Tobler (1969) from manually-digitised DEMs marked another milestone, for it provided the basis for systematising general geomorphometry.

Evans (1972, 1980) criticised the pre-DEM fragmentation of the field (Neuenschwander, 1944), especially its many diverse and unrelated indices calculated or measured by hand from contour maps. Using a manually interpolated DEM and building upon the work of Tobler (1969), Evans (1972) showed that a point (or small x, y neighbourhood) could be characterised by elevation and its surface derivatives slope gradient and curvature, the latter in both plan and profile. Krcho (1973, 2001) independently provided a full mathematical basis for a system of surface derivatives in terms of random-field theory. These parameters could then be summarised for an area by standard statistical measures: mean, standard deviation, skewness, and kurtosis. Following the lead of W.F. Wood, in 1968 Pike and Wilson (1971) began to create USGS' first (manual) DEMs and computer software to calculate an extensive suite of parameters, including the hypsometric integral (Schaber et al., 1979) and values of (apparent) slope and curvature at multiple profile and grid resolutions.

About the same time, Carson and Kirkby (1972) demonstrated the relevance of elevation derivatives to geomorphological (mainly slope) processes, laying the basis for a more mathematical, modelling, approach to geomorphology that was intrinsically quantitative. Measures of surface position and catchment area already had been estimated manually by Speight (1968) to characterise *landform elements*. Pike (1988) subsequently proposed automating the multivariate approach to surface characterisation from DEMs and introduced the concept of the geometric signature of landform types.

Early maps and diagrams of geomorphometric results were limited to low-resolution displays by cathode-ray tube and then to 128 typed characters per line on computer printout-paper 38 cm in width — convenient for tables but clumsy for maps (Chrisman, 2006). With replacement of these crude output devices by pen-driven vector plotters and then high-resolution raster plotters, first in black and finally in colour, computer mapping came of age (Clarke, 1995). Among the most effective displays for topography is the shaded-relief (also *reflectance*) map, which shows the shape of the land surface by variations in brightness. Relief shading originated in the *chiaroscuro* of Renaissance artists. It was highly refined by Imhof (1982) and then automated by his Israeli student Pinhas Yoeli (1967). Comparable techniques[27] are now standard on virtually all GIS and geomorphometric packages. For comprehensive summaries of manual and automated relief shading see http://www.reliefshading.com and Horn (1981).

Computer programs suited to the statistical analysis of topographic data became increasingly available in the 1960s. Particularly useful to the geomorphometrist for sorting out descriptive parameters were techniques of multiple-correlation and factor and principal-components analysis (Lewis, 1968). With the rise of numerical taxonomy in the biological sciences (Sokal and Sneath, 1963) came the complementary multivariate technique *cluster analysis*, wherein observations were

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[27] The first detailed large-format shaded-relief image published as a paper map (Thelin and Pike, 1991) portrayed the conterminous United States from a 12,000,000-point DEM (0.8-km resolution).

automatically aggregated into groups of maximum internal and minimum external homogeneity (Parks, 1966). Cluster analysis proved adept at automating the identification of topographic types and delimiting land-surface regions from samples of land-surface parameters (Mather, 1972).

> REMARK 7. *Development of the digital elevation model (DEM), first publicly described in 1958 by American photogrammetrists at MIT, has paralleled that of the electronic computer.*

Although geomorphometry was taking advantage of the computing revolution[28] in the 1970s and 1980s, limited computer power still held back more ambitious calculations. The constraints on morphometric analysis by 1980s computers are nicely illustrated by Burrough (1986) for a land evaluation project in Kisii, Kenya, where several land-surface parameters were derived from a DEM by the "*Map Analysis Package*" (MAP). Computing capabilities of this pioneering software, developed in FORTRAN by Dana Tomlin at Harvard, were restricted to 60×60 grid cells (see also Figure 9).

A major goal was accurate capture of surface-specific lines from DEMs, the most essential being stream networks. Early efforts at drainage tracing were rather crude: the widely implemented D8 approach routed flow only in eight directions (Figure 7 in Chapter 7), often creating bogus parallel flow lines oblique to the natural ground slope (Jenson, 1985; Jenson and Domingue, 1988). This problem equally reflects inferior DEMs and low-relief topography. Improved methods soon were devised (Fairfield and Leymarie, 1991) to split the flow into adjacent grid cells, yielding more realistic networks, whereupon the *DEM-to-watershed transformation* (Pike, 1995) rapidly grew into an active sub-field that still shows lively development.

By the end of the 1980s, it was possible to process DEMs over fairly large areas. The executable DOS package MicroDEM (Guth et al., 1987), for example, could extract over ten land-surface parameters and visualise DEMs together with remote sensing images. Martz and de Jong (1988), Hutchinson (1989) and Moore et al. (1991a) further advanced hydrological modelling and practical applications in morphometry. Since the early 1990s and the personal computer revolution, algorithms have been implemented in many raster-based GIS packages (see Chapter 10 for a review) and point-and-click geomorphometry on desktop and laptop machines is now the everyday reality.

3.4 The quantification of landforms

Recognition and delimitation of such discrete features as *drainage basins* (Horton, 1932, 1945), *cirques* (Evans, 2006), *drumlins* (Piotrowski, 1989), and *sand dunes* (Al-Harthi, 2002) on a continuous surface is more difficult than that of elementary forms and thus *Specific Geomorphometry* remains the more subjective practice

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[28] Mark (1975a, 1975b), Grender (1976), and Young (1978) were among the pioneers who developed operational programs to calculate slope, aspect, and curvatures from gridded DEMs. See also Schaber et al. (1979), Horn (1981), and Pennock et al. (1987).

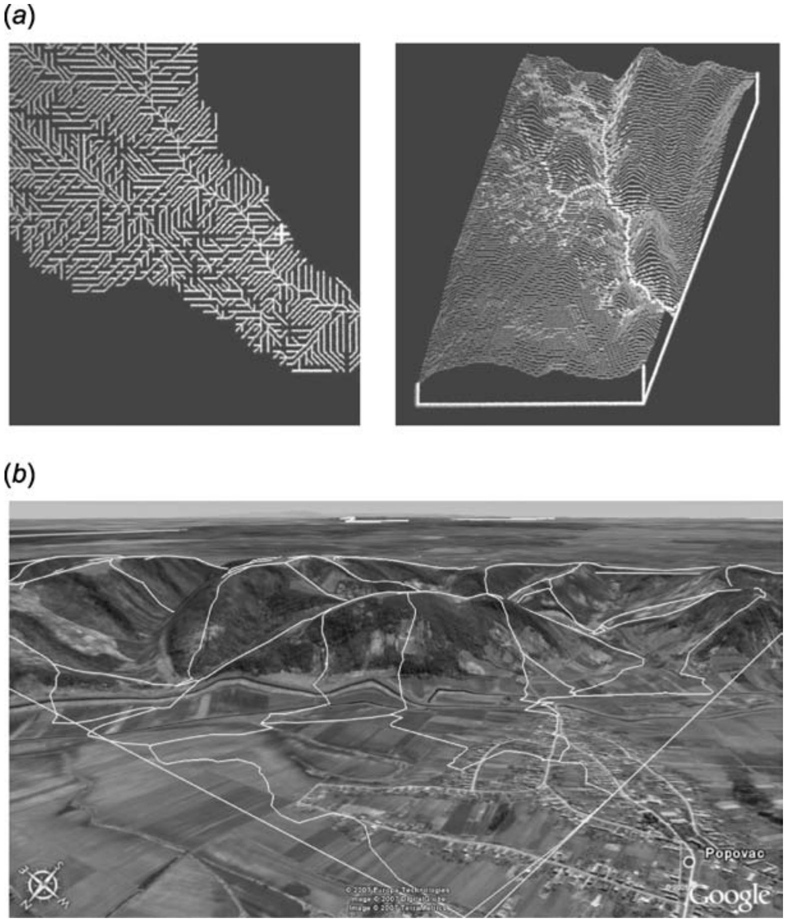

FIGURE 9 Geomorphometry then and now: (a) output from late-1980s DOS programme written to display land-surface properties: (left) map of local drainage direction, (right) cumulative upstream drainage elements draped over a DEM rendered in 3-D by parallel profiles. Courtesy of P.A. Burrough; (b) watershed boundaries for the Baranja Hill study area overlaid in Google Earth, an online geographical browser accessible to everyone. (See page 708 in Colour Plate Section at the back of the book.)

(Evans and Cox, 1974). While this book does not delve deeply into this area (Evans, 1972; Jarvis and Clifford, 1990), it warrants brief mention here.

Astronomy was the first science to quantify, so it is no surprise that the earliest scientific measurement of a landform involved not Earth but the craters on its Moon (Pike, 2001b). An impact crater is rather easy to distinguish from the surrounding land surface and its axial symmetry enables its shape to be captured completely by only a few simple parameters. Not all landforms are so favoured; alluvial fans, landslides, dolines, and other features all require good operational definitions to ensure their proper characterisation. The introduction of DEMs has not eased this requirement, and the added precision (not necessarily accuracy!)

comes at the cost of measurement complexity (Mouginis-Mark et al., 2004). While the automated definition of, say, *valleys* and *valley heads* from DEMs can be tested against their visual recognition (Tribe, 1991, 1992b), the low accuracy of many DEMs can spoil such an exercise (Mark, 1983). Regardless, more Earth scientists are now using DEMs as their primary source of data for landform measurement (e.g. Walcott and Summerfield, 2008).

4. GEOMORPHOMETRY TODAY

DEM-based geomorphometry continues to evolve from a number of the themes described above. Geostatistical analysis has established spatial autocorrelation, quantification of the 'First Law of Geography' — "*Everything is related to everything else, but near things are more related than distant things*" (Tobler, 1970) — as a routine technique (Bishop et al., 1998; Iwahashi and Pike, 2007). Fractional dimensionality (Mandelbrot, 1967) and self-similarity (Peckham and Gupta, 1999) still appear to be useful for representing drainage networks and other spatial phenomena, although their extension to land-surface relief z thus far has been modest (Klinkenberg, 1992; Outcalt et al., 1994). Multi-resolution modelling of the land surface is a vital topic of study (Sulebak and Hjelle, 2003), and recent analysis of fluvial networks on Mars continues to extend the utility of DEMs (Smith et al., 1999). Further examples of contemporary geomorphometry will be found in the following chapters of this book, especially by way of software development in Part II and their applications in Part III.

The maturing of GIS and remote-sensing technology has enabled geomorphometry to emerge as a technical field possessing a powerful analytical toolbox (Burrough and McDonnell, 1998). At the outset of the 21st century, geomorphometry is not only a specialised adaptation of surface quantification (mainly geometry and topology) to Earth's topography, but an independent field comparable to many other disciplines (Pike, 1995, 2000a).

With today's rapid growth in sources of mass-produced DEMs, such as the Shuttle Radar Topographic Mission (SRTM) and laser ranging (LiDAR) surveys (see also Chapter 3), land-surface parameters are finding ever-increasing use in a number of areas. These range from precision agriculture, soil–landscape modelling, and climatic and hydrological applications to urban planning, general education, and exploration of the ocean floor and planetary surfaces. Earth's topography has been sufficiently well sampled and scanned that global DEM coverage now is available at resolutions of 100 m or better. Good DEM coverage is available beyond Earth. In fact, among Solar System planets, Mars has the most accurate and consistent DEM, with vertical accuracy up to ±1 m (Smith et al., 1997; Pike, 2002).

Geomorphometry has become essential to the modelling and mapping of natural landscapes, at both regional and local scales (see further Chapter 19). Applications in the restricted sense of parameter and object extraction are distinguished from the use of DEMs for landscape visualisation or change detection. All varieties of spatial modelling are available, stochastic (e.g. spatial prediction) as well as process-based (e.g. erosion modelling). Because land-surface parameters and objects are now relatively inexpensive to compute over broad areas of interest, they can be used — with due caution — to replace *some* of the boots-on-the-ground field sampling that is so expensive and time-consuming.

REMARK 8. *Geomorphometry supports Earth and environmental science (including oceanography and planetary exploration), civil engineering, military operations, and video entertainment.*

The many uses of geomorphometry today can be grouped into perhaps five broad categories:

Environmental and Earth science applications Land-surface parameters and objects have been used successfully to predict the distribution of soil properties (Bishop and Minasny, 2005), model depositional/erosional processes (Mitášová et al., 1995), improve vegetation mapping (Bolstad and Lillesand, 1992; Antonić et al., 2003), assess the likelihood of slope hazards (Guzzetti et al., 2005), analyse wildfire propagation (Hernández Encinas et al., 2007), and support the management of watersheds (Moore et al., 1991a). Geomorphometric analyses further aid in deriving soil–landscape elements and in providing a more objective basis for delimiting ecological regions. Recent developments include automated methods to detect landform facets by unsupervised fuzzy-set classification (Burrough et al., 2000; Schmidt and Hewitt, 2004). Land-surface parameters even play a role in automatically detecting geological structures and planning mineral exploration (Chorowicz et al., 1995; Jordan et al., 2005).

Civil engineering and military applications Both fields were early users of DEMs (Miller and Laflamme, 1958). Today, engineers frequently employ DEM calculations to plan highways, airports, bridges, and other infrastructure, as well as to situate wind-energy turbines, select optimal sites for canals and dams, and locate microwave relay towers to maximise cell-phone coverage (Petrie and Kennie, 1987). Li et al. (2005, §14) review recent applications. Land-surface quantification is crucial to any number of military activities (Griffin, 1990); DEMs are used to simulate combat scenarios, actively guide ground forces as well as terrain-following missiles, and to automate line-of-sight and mask-angle calculations for concealment and observation (Guth, 2004; http://terrainsummit.com). Viewshed algorithms operating on DEMs have been found superior to simplistic sightline analysis for siting air-defence missile batteries (Franklin and Ray, 1994). As in the past (see above), much defence-related geomorphometry is classified and thus unavailable to the wider scientific community.

Applications in oceanography Measurement of seafloor topography is the province of *bathymetry*. DEMs — or rather DDMs (*digital depth models*[29]) — of the seafloor figure prominently in coastal geomorphology, geophysical analysis of global tectonics, the study of ocean currents, design of measures to protect shorelines from erosion, mineral exploration, and fisheries management (Burrows et al., 2003;

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[29] See http://dusk2.geo.orst.edu/djl/samoa/ for an example of an archive of GIS data from multibeam bathymetry and submersible dives supporting a marine sanctuary in Samoa.

Giannoulaki et al., 2006). Surface parameters and objects computed for the seabed from DDMs have been used to optimise fish farming and to improve the mapping of marine benthic habitats (Bakran-Petricioli et al., 2006; Lundblad et al., 2006). Finally, seafloor morphometry plays a critical role in the navigation and concealment of nuclear submarines.

Applications in planetary science and space exploration A scientific understanding of Earth's Moon and the solid planets increasingly depends upon DEMs. LiDAR data from the 1994 Clementine[30] mission to the Moon produced two broad-scale global DEMs (Smith et al., 1997); their modest spatial resolutions of 1 and 5 km revealed previously unknown giant impact scars (Williams and Zuber, 1998; Cook et al., 2000). Grid resolution of the global DEM resulting from the spectacularly successful 1998–2001 Mars Orbital Laser Altimeter (MOLA[31]) mission exceeds that of Earth[32] (Smith et al., 1999)! Geomorphometry is well suited to take advantage of these results, as demonstrated by Dorninger et al. (2004) and by Bue and Stepinski (2006), who used the MOLA DEM to test algorithms for the automated recognition of landforms.

Applications in the entertainment business Mass-produced DEMs are essential to video game and motion picture animation, where geomorphometry is referred to as *terrain rendering*[33] (Blow, 2000). Usually structured in TIN arrays, these DEM applications range from creating background scenery to simulating landscape evolution and modelling sunlight intensity (often using Autodesk's 3ds Max package). Pseudo-realistic rendering is sufficient to create a visually convincing product, so exact reproduction of real-world landscapes is rarely necessary. Because the industry is highly competitive, design teams do not always publish their methods, making it difficult to follow the latest innovations.

Not all applications of geomorphometry are well developed or supported. Terrain rendering for computer games, for example, commands more financial resources than all environmental land-surface modelling combined (Pike, 2002)! Other generously-funded areas in the past have included military operations and space exploration. Any soil- or vegetation-mapping team would be grateful for the access to technology and data available to game developers or military surveillance agencies.

5. THE “BARANJA HILL” CASE STUDY

To enhance understanding of the algorithms demonstrated in Part II of this book, we will use a small case study consistently[34] throughout. In this way, you will be able to compare land-surface parameters and objects derived from different

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[30] http://pds-geosciences.wustl.edu/missions/clementine/.

[31] http://wwwpds.wustl.edu/missions/mgs/megdr.html.

[32] The current global Mars DEM is at resolution of 1/128 of a degree, which at the equator is about 460 m. Locally, resolution is much better than that.

[33] See also the http://vterrain.org project.

[34] We were inspired mainly by statistics books that demonstrated several processing techniques on the same dataset, such as Isaaks and Srivastava (1989).

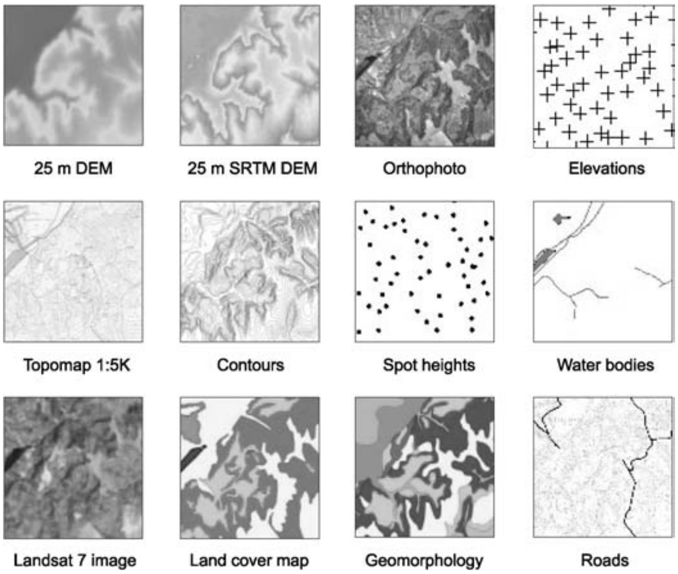

FIGURE 10 The "Baranja Hill" datasets. Courtesy of the Croatian State Geodetic Department (http://www.dgu.hr). (See page 709 in Colour Plate Section at the back of the book.)

algorithms and software packages and thus more easily find the software best suited to your needs.

The "*Baranja Hill*" study area, located in eastern Croatia, has been mapped extensively over the years and several GIS layers are available at various scales (Figure 10). The study area is centered on 45°47′40″N, 18°41′27″E and corresponds approximately to the size of a single 1:20,000 aerial photo. Its main geomorphic features include hill summits and shoulders, eroded slopes of small valleys, valley bottoms, a large abandoned river channel, and river terraces (Figure 11).

The Croatian State Geodetic Department provided 50k- and 5k-scale topographic maps and aerial photos (from August 1997). An orthorectified photo-map (5-m resolution) was prepared from these source materials by the method explained in detail by Rossiter and Hengl (2002). From the orthophoto, a land cover polygon map was digitised using the following classes: agricultural fields, fish ponds, natural forest, pasture and grassland, and urban areas. Nine landform elements were recognised: summit, hill shoulder, escarpment, colluvium, hillslope, valley bottom, glacis (sloping), high terrace (tread) and low terrace (tread).

Contours, water bodies, and roads were digitised from the 1:50,000 and 1:5000 topographic maps. Contour intervals on the 1:50,000 topographic map are 20 m in hill land and 5 m on plains, and on the 1:5000 map they are 5 and 1 m respectively. From the 1:5000 contours and land-survey point measurements, a 5 m DEM was derived by the ANUDEM (TOPOGRID) procedure in ArcInfo (Hutchinson, 1989), and then resampled to a 25 m grid. For comparison, the 30 m SRTM DEM (15′ × 15′ block) obtained from the German Aerospace Agency (http://eoweb.dlr.de) was resampled to 25 m (Figure 6 in Chapter 3). The total area of the case study is

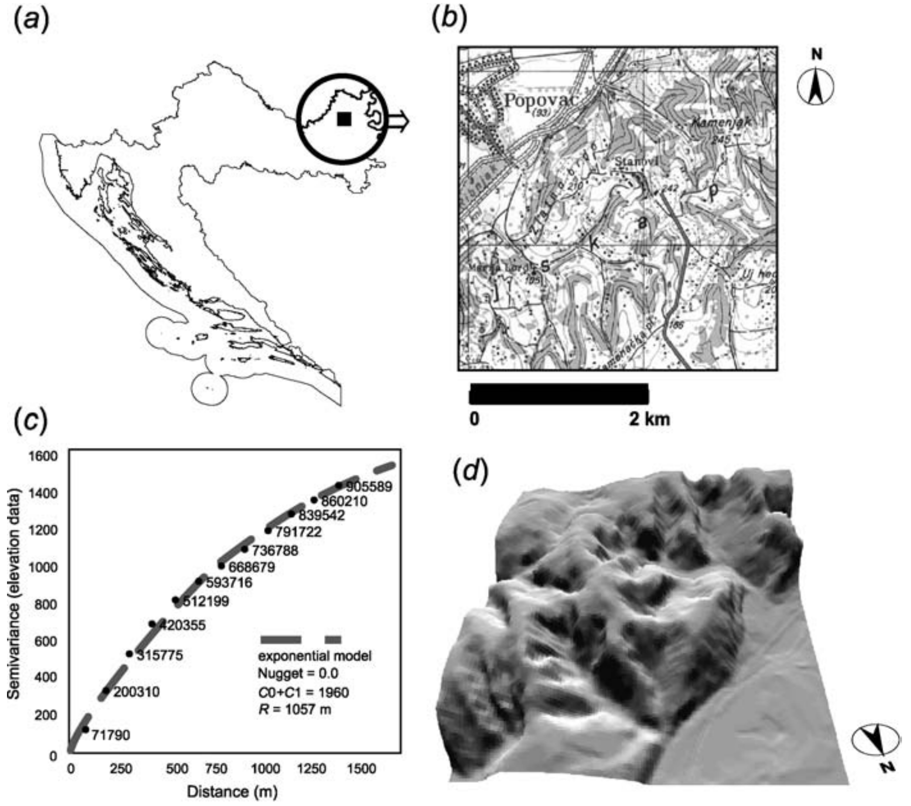

FIGURE 11 The "*Baranja Hill*" study area: (a) location in eastern Croatia; (b) 1:50,000 topographic map (reduced) showing main features; (c) omnidirectional variogram from the elevation point data; and (d) perspective view of the area. Courtesy of State Geodetic Administration of Republic of Croatia.

13.69 km^2 or 3.6×3.7 km. Elevation of the area ranges from 80 to 240 m with an average of 157.6 m and a standard deviation of 44.3 m. Both 25-m DEMs have been brought to the same grid definition with the following parameters: `ncols` = 147, `nrows` = 149, `xllcorner` = 6,551,884, `yllcorner` = 5,070,562, `cellsize` = 25 m. We used the local geodetic grid (Croatian coordinate system, zone 6) in the Transverse Mercator projection on a Bessel 1841 ellipsoid ($a = 6{,}377{,}397.155$, $f^{-1} = 299.1528128$). The false easting is 6,500,000, central meridian is at 18° east, and the scale factor is 0.9999. Note also that, to have proper geographic coordinates, you will need to specify a user-defined datum of $\Delta X = 682$ m, $\Delta Y = -199$ m and $\Delta Z = 480$ m (Molodensky transformation). The projection files in various formats are available on this book's website. The complete "Baranja Hill" dataset[35] consists of (Figure 10):

`DEM25m` 25-m DEM derived from contour lines on the 1:5000 contour map;

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[35] You can access the complete "Baranja Hill" dataset via the geomorphometry.org website.

`DEM25srtm` 25-m DEM from the Shuttle Radar Topographic Mission;

`DEM5m` 5-m DEM derived from stereoscopic images;

`contours5K` Map of contours digitised from the 1:5000 topo-map;

`elevations` Point map ($n = 853$); very precise measurements of elevation from the land survey;

`wbodies` Layer showing water bodies and streams;

`orthophoto` Aerial (orthorectified) photo of the study area (pixsize = 5 m);

`satimage` Landsat 7 satellite image with 7 bands from September 1999;

`landcover` Land-cover map digitised from the orthophoto;

`landform` Polygon map of the principal landform elements (facets);

`fielddata` Field observations at 59 locations are available in report form.

6. SUMMARY POINTS

Geomorphometry is the science of quantitative land-surface analysis. A mix of Earth and computer science, engineering, and mathematics, it is a new field paralleling analytical cartography and GIS. It evolved directly from geomorphology and quantitative terrain analysis, two disciplines that originated in 19th-century geometry, physical geography, and the measurement of mountains.

Classical morphometry (orometry) was directed toward hypsometry and plan form, and calculating average elevation and slope, volume, relative relief, and drainage density from contour maps. Later work emphasised drainage topology, slope-frequency distribution, and land-surface classification. Techniques have ranged from trend-surface and spectral analysis of surveyed elevations and profiles to geostatistical and fractal analysis of 3-D elevation arrays.

Modern geomorphometry addresses the refinement and processing of elevation data, description and visualisation of topography, and a wide variety of numerical analyses. It focuses on the continuous *land surface*, although it also includes the analysis of *landforms*, discrete features such as watersheds. The operational goal of geomorphometry is extraction of measures (*land-surface parameters*) and spatial features (*land-surface objects*) from digital topography.

Input to geomorphometric analysis is commonly a *digital elevation model* (DEM), a rectangular array of surface heights. First described in 1958, DEMs developed along with the electronic computer. Many DEMs are prepared from existing contour maps; because all DEMs have flaws and even advanced technologies such as LiDAR introduce errors, DEMs must be corrected before use. The growth in sources of mass-produced DEMs has increased the spread of geomorphometric methods.

Geomorphometry supports countless applications in the Earth sciences, civil engineering, military operations, and entertainment: precision agriculture, soil–landscape relations, solar radiation on hillslopes, mapping landslide likelihood, stream flow in ungauged watersheds, battlefield scenarios, sustainable land use, landscape visualisation, video-game scenery, seafloor terrain types, and surface processes on Mars.

Geomorphometric analysis commonly entails five steps: sampling a surface, generating and correcting a surface model, calculating land-surface parameters or objects, and applying the results. The three classes of parameters and objects (basic, hydrologic, and climatic/meteorological) include both landforms and point-measures such as slope and curvature. Landform *elements* are fundamental spatial units having uniform properties. Complex analyses may combine several parameter maps and incorporate non-topographic data.

The procedure that extracts most land-surface parameters and objects from a DEM is the *neighbourhood operation*: the same calculation is applied to a small sampling window of gridded elevations around each DEM point, to create a complete thematic map. Processing is simplified by the *raster* (grid-cell) structure of the DEM, which matches the file structure of the computer. Because parameters can be generated by different algorithms or sampling strategies, and vary with spatial scale, no DEM-derived map is *definitive*. To encourage readers to compare maps created by the different software packages demonstrated in this book, several digital datasets for a small test area (Baranja Hill) are available via the book's website.

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http://geomorphometry.org — the geomorphometry research group.