Mapping gradient fields of landform migration

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Abstract — Geospatial analytics techniques describing changes of unstable landscapes provide critical information for hazard management and mitigation. We propose a method for quantifying horizontal migration of complex landforms based on the analysis of contour evolution. When applied to a set of elevations this technique provides comprehensive information on magnitude and direction of landform migration at any point in space and time. The method is based on the concept of space-time cube combined with GIS-based analysis applied to spatio-temporal surface. The result of the analysis is a vector field representing the movement and deformation of contours. We also present several approaches to visualization of these vector fields as space-time gradient lines, vectors or dynamic “comets”. We demonstrate the method on a laboratory model and an elevation time series capturing evolution of a coastal sand dune.

I. INTRODUCTION

Natural hazards often involve significant changes in topography induced by coastal or stream channel erosion, aeolian sand transport, or gravitation forces on unstable hillslopes. Quantification of these changes, especially their evolution over time, is critical for hazard management and mitigation. Modern 3D mapping technologies such as lidar are now routinely used to monitor 3D landscape change at high spatial and temporal resolutions. Over the past decade new methods and techniques were developed to analyze these monitoring data and derive quantitative metrics of observed changes [1, 2]. DEM differencing, per cell statistics, as well as aggregated metrics such as total volume change can be computed using standard raster analysis tools [3]. Time series of DEMs can also be visually analyzed using dynamic 3D techniques implemented in GIS [4].

Horizontal migration rates of landforms which involve change in landform geometry (as is often the case with dunes and shorelines) are harder to quantify because the rates are spatially variable and involve change in both magnitude and direction. Standard techniques for assessment of line feature migration rates are based on transects approximating perpendicularly to the direction of feature migration and on measurement of displacement along these transects [3, 5]. This approach is limited by the transect spacing and does not provide information on migration direction change. Also, dramatic changes in landform can make it difficult to generate valid transects.

We propose a method for quantifying horizontal migration of complex landforms based on the analysis of contour time series with the aim to generate a quantitative representation of magnitude and direction of landform evolution at any point in space and time.

II. APPROACH

Landscape evolution is often represented by a time series of DEMs derived from repeated 3D surveys, often using lidar technology. To analyze horizontal migration and deformation of landforms within a dynamic landscape we introduce the following concept. Given a time series of n DEMs we can represent the evolution of landscape in space time cube (STC) where the third coordinate is time $t$ and the modeled variable is elevation $z$:

$$\text{z} = f(x, y, t).$$

Landform evolution at a given constant elevation $z = c$ can then be represented and visualized as an isosurface derived from the STC representation (Fig. 1).

To quantify the rate and direction of contour horizontal migration we can segment the time series of contours $z_i = c$, $i = 1, \ldots, n$ into non-intersecting segments. Each of these sets of contour segments then define a bivariate function $g_c$ which represents time $t$ as a function of contour position $(x, y)$:

$$t = g_c(x, y).$$

1The mathematical definitions of contour and isosurface are the same since both are special cases of a level set which is defined as $f(x_1, \ldots, x_n) = c$ or more precisely as $L_c = \{(x_1, \ldots, x_n) \mid f(x_1, \ldots, x_n) = c\}$.

2There should be no other contour between two successive states of one contour, i.e., following the surface in the direction of increasing time, we first get to a newer state of the contour we started from before any other contour. This is equivalent to segmentation of the isosurface in Fig. 1 into sub-surfaces which can be represented by bivariate functions.
Additionally, only the areas between the contours in successive times are considered and these areas must fulfill the following condition:

\[ f(x, y, t_i) > c \Delta f(x, y, t_{i+1}) > c \]  \hspace{1cm} (3)

where \( \Delta \) is a symmetric difference of two sets defined as \( A \Delta B = (A \cup B) \setminus (A \cap B) \).

The time series of contour segments which fulfill the above condition can then be interpolated using a suitable GIS-based interpolation to create a raster representation of the temporal function \( g_c(x, y) \). This function then allows us to derive a vector field describing the movement of a contour by computing its gradient:

\[
\nabla g_c = (g_x, g_y), \quad \text{where } g_x = \frac{\partial g_c}{\partial x}, \quad g_y = \frac{\partial g_c}{\partial y}.
\]  \hspace{1cm} (4)

For visualization in GIS, it is convenient to represent gradient using its direction \( \theta \) (aspect) and magnitude \( w \) (slope) components:

\[
\tan \theta = \frac{g_y}{g_x}, \quad w = \sqrt{g_x^2 + g_y^2}.
\]  \hspace{1cm} (5)

The gradient vector field is then represented as two raster maps \( (w \) and \( \theta \). Since gradient magnitude \( w[\text{time/length}] \) of the temporal function \( t = g_c(x, y) \) is an inverse value of rate of change we compute the speed (rate) of horizontal migration \( v[length/time] \) as:

\[
v = \frac{1}{w}.
\]  \hspace{1cm} (6)

In other words, if two contours from two consequent time snapshots are spatially close to each other, this will lead to steep slope (large \( w \)) in \( g_c \) and a low horizontal migration speed.

Now we have a two-dimensional vector field which assigns a vector defined by direction \( \theta \) and speed \( v \) to each position \( (x, y) \). This vector field represents the rate and direction of landform migration at a given elevation \( c \). We can derive such a vector field for a set of elevations representing the entire landform and obtain a 3D, spatially variable representation of its horizontal migration and deformation. We can also map locations of migration acceleration and rate of deformation by computing relevant metrics based on second order derivatives (divergence of the vector field or spatio-temporal "profile" curvature).

To support the presented concept, we have used and further developed visualization techniques for graphical representation of vector fields using gradient lines, arrow fields, and dynamic comet-like visualization [4]. The raster maps representing migration rates at multiple elevations can also be stacked into a 3D raster (voxel model) and areas of equal migration rates can be extracted and visualized as isosurfaces.

III. APPLICATIONS

We are exploring application of the presented technique to the mapping of migration vector fields associated with various types of landscapes and processes. Here we present a test of the algorithm using laboratory models and a real-world application for analysis of a coastal sand dune migration.

A. Laboratory experiment

We have used a laboratory terrain model to test our methodology and algorithms in a fully controlled environment. Our tangible geospatial modeling system Tangible Landscape [6], allows us to create realistic terrain models from polymeric sand in a relatively intuitive way while providing real-time feedback about our model properties using contours, slope or flow pattern (Fig. 2).

The initial model and a sequence of its modifications was scanned and imported into GIS providing a series of DEMs suitable for testing the performance of our algorithms for different
landform geometries. Our test case was designed in such a way that the hill migrated in one prevailing direction while changing its shape. For this type of migration, we can compute the vector field without segmentation of the contour time series.

The example illustrates the landform migration analysis using 4 different states with the assigned range of elevation values between 103 m and 128 m. The individual states were assigned the years 2001, 2005, 2008, and 2009 so the time interval varied from 1 to 4 years. The resulting migration rate and direction at the elevation \( z = 110 \) m was visualized by gradient lines, vector arrows (Fig. 3), and a comet-like visualization.\(^3\) The comet-like visualization tool was modified so that the comets are generated and move only in relevant areas, while the 4 states of terrain represented by a series of elevation maps are periodically changing in background.

**B. Jockey’s Ridge sand dunes**

We have applied the method to measure migration of the Jockey’s Ridge sand dune field located in a state park on the North Carolina coast. The dunes have been migrating at variable rates with sand often transported outside the park boundaries, obstructing roads and threatening homes in neighboring communities. The approximate rate of migration was assessed for the first time several years ago from a series of DEMs by manually measuring distances between consecutive positions of dune crests [7]. The process was time consuming and to some extent subjective because the distance between the crests was highly variable as the dune has changed its shape and elevation.

We have used the presented method to analyze the spatial pattern of dune migration, including the dune windward side which was not measured previously using the crest-based method. We also measured and compared horizontal migration rates at different elevations—here we present the migration gradient fields at elevations 10, 12, 14 and 16 m based on a time series of DEMs representing the dunes in the 1974, 1995, 2001 and 2008 years (see [3] and [7] for description of data and processing including correction of registration errors). As expected, the resulting gradient field shows a more homogeneous pattern at lower elevation (10 m) compared to higher variability in both rates and direction at higher elevation (16 m). The analysis also reveals a relatively stable pivot point, around which the dune migration changes its direction. The migration rates presented here for the windward side of the dune are comparable to the values at the leeward side, estimated manually from the crests [7] but the vector field provides much more detailed information about the spatial variability of the migration and the mapping process is to a large extent automated.

**IV. DISCUSSION AND FUTURE WORK**

It is important to note that the migration vector field does not represent the physical transport of the soil or sand particles. Instead, it provides information how a landform geometry at the given elevation was transformed between the time snapshots due to the redistribution of its mass. Such information can be used not only for dune management but also to improve dune evolution simulations by deriving more accurate relationships between the elevation and sand transport [8].

\(^3\)Comet-like visualization is available online at http://ncsu-osgeorel.github.io/spatio-temporal-contour-evolution.
V. CONCLUSION

We implemented the presented algorithm for the computation of horizontal migration vector fields from spatio-temporal sets of contours in a GRASS GIS module *r.contour.evolution*. The module input is a series of DEMs and elevation values, the output is a set of raster maps which represent migration gradient field and its properties. The presented method further extends the set of tools for analysis of evolving topography outlined in [2, 3], by providing a more detailed and automated approach for assessment of horizontal migration of dynamic landforms.

REFERENCES


