Land surface derivatives: history, calculation and further development.

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Abstract— **The use of derivatives in geomorphometry is reviewed. For first and second derivatives (slope and curvatures) of the land surface, represented by a square-grid DEM, use of a quadratic based on nine points is recommended. For detailed analysis of slope gradient, maximum slope (based on two points) may be useful: but only if the DEM is very accurate. For curvature, streamline (rotor) curvature should be added to profile and plan curvature. The system of local point-based variables in the gravity field is now extended to third derivatives.**

1. INTRODUCTION: HISTORY

The importance of slope gradient in geomorphological processes and the description of land surface form has long been recognized (Young, 1972; Parsons, 1988). Traditionally it was measured directly in the field along profiles, using clinometers, levels or pantometers. This involved problems of sampling (Young, 1972) and the ground length over which it is to be measured: Pitty (1969) favoured the fixed slope length given by a 1.52 m pantometer, giving a 'human scale'. Also, subtle differences can be observed between up-slope and down-slope measurements. Methods of measuring curvature tangential to the slope profiles were developed by Young (1972). Slope gradient was measured also from contour spacing on maps, and generalized maps of gradient were produced from contour density, e.g. counting contour intersections with a grid.

The calculation of derivatives from gridded height data was pioneered by geophysicists (e.g. Šalamon, 1963). This was applied to gridded models of the land surface by Tobler (1969). Geophysicists also began consideration of the land surface as a scalar field, and systematic application of methods of mathematical analysis, a significant theoretical shift which was developed further by Krcho (1973). Drawing on the earlier work of Tobler and W.A. Wood, Evans (1972) demonstrated the value of derivatives in geomorphology, and attempted to simplify at-a-point geomorphometric variables in terms of two components (gradient, aspect) of the first derivative and two (plan and profile curvature) of the second. He showed their sensitivity to scale, represented by DEM grid mesh. Further work has dealt with various measurement scales or transformations for gradient and curvatures as distributed over areas. The value of all these derivatives has been amply confirmed, and their importance increases as new applications are found. Mitášová and Hofierka (1993) redefined plan and profile curvatures on the base of differential geometry theory (inverting the geomorphological sign convention), and more components of curvature have been added (Jenčo, 1992; Shary et al. 2002). Numerous variant definitions of curvature have complicated matters, reducing the simplification originally desired. Nevertheless, consistency of definition is highly desirable for a science to advance by making measurements which are comparable.

2. CALCULATION OF SLOPE (GRADIENT AND ASPECT)

Currently there is competition as to the value of different definitions, and debate about how variation with scale (or degree of generalisation) should be handled. For gradient calculation from gridded DEMs, I suggest that both very local (D8: 2-point, 8-direction, maximum-slope), and slightly smoothed (9-point quadratic, omni-directional) values, are valuable – assuming data are sufficiently accurate. Onorati et al. (1992) illustrated differences between three methods for slope, for a 230 m grid of Italy, and chose a 3-point method. Zhou and Liu (2004) compared six methods for two mathematical surfaces, and demonstrated the desirability of using 8 points rather than 4 or 3: weighting by distance made little difference.

Although 2-point gradients (using steepest descent from the central point to any of the eight 'Queen's case' neighbours) are very sensitive to data error, they do make full use of DEM detail and preserve the range of values, which is reduced by all methods that use more than 2 points. Conventionally the gradient is attributed to the source (central, upslope) grid point: if it were placed accurately, half-way between the two points, the resulting gradient values would be unevenly distributed, on an incomplete grid (with many holes). Fitting exact planes to each set of three adjacent points makes all directions (aspects) possible, rather than just eight 'cardinal points' of the compass, but the results relate to new points in the centres of the two sets of triangles produced from a square grid. Fitting planes to four points already involves some generalization of the DEM, and the results relate to new points displaced by half the grid mesh. Fitting to five points (a central point and four closest neighbours, 'rook's case') increases generalization but does provide results at each of the original data points.

3. CALCULATION OF SLOPE AND CURVATURES

None of these (2, 3, 4 or 5 point algorithms) gives reliable estimates of surface curvature. This is why Evans (1980) adopted a full quadratic (6-parameter) equation fitted to nine points (3 x 3). Tests by Skidmore (1989), Eyton (1991), Guth (1995), Florinsky (1998) and Schmidt et al. (2003) have demonstrated the advantages of this method over several alternatives, even though it inevitably smooths sharp breaks. Wise (1998) also showed the advantages of the nine-point quadratic, especially for aspect estimation (n.b. slope is estimated from 8 points: the central point is used only for curvatures). The quadratic provides better results for gradient (Florinsky, 1998) and for curvatures (Schmidt et al., 2003) than a 9-parameter partial quartic. Once error in the DEM is admitted, it seems undesirable to constrain the surface to pass through the central point, as this is affected by error as much as are the other eight. There may, however, be a case for weighting the central point more, and the corner points less, than the other four. Conversely, some algorithms use eight neighbours and ignore the central point, which seems perverse.

Shary et al. (2002) have suggested routinely fitting over 5 x 5 points, to smooth the errors in contour-based DEMs. The degree of smoothing should be related to the degree of DEM error: I suggest basing this on the ratio of standard error of altitude to the mean difference between adjacent points. We await calibration of the desirable degree of smoothing as a function of this ratio: it is probably best to smooth as necessary first, before fitting the nine-point quadratic.

Guth (1995) demonstrated that use of all eight neighbours reduced mean gradients to 78% of those from the 'steepest adjacent neighbour' (i.e. two-point) algorithm. This is roughly equivalent to the reduction when grid mesh is doubled.

Reference to the direction of gravity[1] distinguishes the use of derivatives in earth sciences, as distinct from broader mathematical schemes, although Shary et al. (2002) have demonstrated the use of principal curvatures. Evans (1980) used the 6-parameter quadratic as the basis for implementation of a 5-variable system of local point-based variables in the gravity field: altitude, slope gradient, slope aspect, profile curvature and plan curvature. Inevitably this attempt to simplify general geomorphometry has been followed by extension to greater complexity (Olaya, 2009), and it is clear that several additions are necessary (Evans and Minár, 2011). Improved computing capabilities permitted the very desirable extension to flow-line related positional variables (not dealt with here). For

[1] We generally assume that a land surface can be represented as a (single-valued) function, z = f(x,y), with the z-axis parallel to the direction of gravity (e.g. Mitášová & Hofierka, 1993).

local variables, the most obvious omission was the third curvature variable: curvature of any surface in 3 dimensions can be completely characterised by three orthogonal variables, but there are many alternative trios of definition. If we keep profile and plan curvature (because of their clear relation to surface processes), the third curvature may be termed ‘rotor curvature’ (Florinsky, 1998) or ‘streamline curvature’ and describes the curvature of streamlines or the divergence of contour lines (Shary et al., 2002; Peckham, 2011). Tangential curvature, in a plane orthogonal to the surface and the flow line, is less useful because it is closely related to plan curvature (Peckham, 2011), especially on low gradients.

4. TRANSFORMATIONS OF FREQUENCY DISTRIBUTIONS

For most types of statistical analysis, it is important to check the shape of the frequency distribution of each variable. These vary between areas, even between adjacent areas (Minár et al. 2013). The venerable and developing literature on hypsometry shows that altitude can be right-skewed or left-skewed, so no single transformed scale will fit all study areas. Slope gradient in degrees or tangents is more often right- (positively-) skewed, so logarithmic transformation is appealing (Speight, 1971). This is because, even in mountain regions, deposition in fans, floodplains and lakes produces extra areas of low gradient. Where these are absent, however, distributions may be symmetrical or, where high relief pushes gradient toward a limiting value for slope stability, negatively skewed (Oguchi et al., 2011).

For real-world DEMs, the distribution of curvatures measured in degrees per unit length (100 m in Evans 1980) is always strongly peaked at the mode of zero. The presence of extremely positive and negative values can greatly bias calculation of product-moment correlations. (In profile convexity these extremes represent sharp breaks and concentrate in high-gradient areas; in plan convexity, they are not just across sharp channels and ridges but also on floodplains, where aspect is almost indeterminate.) To solve this, I have applied arctangent transformations to ‘bring in’ both tails. Unfortunately these require calibration of a constant multiplier, k:

Transform = arctan (k . convexity)

k is chosen so as to minimise kurtosis, originally a rather clumsy trial-and-error process. Note that kurtosis on the negative side of normal (toward a rectangular distribution) is not usually a problem, as it is outliers or extreme values (long tails) that bias correlations and related statistics. Alternatives to the arctangent are the use of other sets of curvatures (would tangential or maximum and minimum curvature avoid the floodplain problem?) or of robust statistics, but it is hard to find applications of those approaches.

5. THIRD DERIVATIVES

First and second derivatives seemed adequate to cover applications in geomorphology up to 1980, but the work of Florinsky (2009) and of Minár et al. (2013) has now demonstrated several applications for the third derivative. Data unreliability has been the main deterrent to their use as yet, but this is improving and better computation techniques have been devised. These third derivatives are valuable in delimiting surface objects (such as elementary forms, forms homogeneous in one or more derivatives) and testing their tendency to a constant value of some derivative. Minár et al. (2013) show that as higher derivatives are taken, they concentrate increasingly around zero, as predicted by the concept of elementary forms. The increase is greater than for random or ‘fractal’ surfaces. Note, however, that as further derivatives of real land surfaces are taken, the resulting surfaces are rougher and rougher, unlike those of mathematically-defined polynomial surfaces. One promising application of third derivatives arises because, while zero values of tangential changes are widespread for both profile and tangential curvature, they coincide only on (sharp) ridges and valleys and both conditions are needed in delimiting these.

6. CONCLUSION

The system of first and second derivatives of land surface altitude in the gravity field has proved robust, popular and useful over the last four decades, and extended to the third derivative it should remain a cornerstone of general geomorphometry for decades to come. Applications are legion (Hengl and Reuter, 2009), and increasing as DEMs at the fine scales relevant to surface processes become available. Uses for field-invariant variables, a further extension, have been proposed by Shary et al. (2002) and we hope to see numerous applications in future. Alternative systems for the analysis of rough surfaces, such as spectral and fractal analyses, are more difficult to implement in practice and to interpret. In fact our ability to differentiate representations of the land surface suggests that it cannot be truly fractal, as fractal surfaces are non-differentiable. With their more direct meaning – rates of change – derivatives remain intuitively appealing.

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