

# Scale-Specific Modeling of Class-Level Uncertainty in Landform Taxonomies Using Fuzzy Sets

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**Abstract**— A multi-scale geomorphometric landform system was created through the use of fuzzy semantic import models and fuzzy overlay to measure distribution of landforms within parcels of the Conservation Reserve Program in Northeast Kansas, United States. The uncertainty and stability of landform classes was measured by calculating the area proportions covered by these classes at varying levels of classification entropy across different scales. Within each scale (defined here as search radius), the landform classes *backslopes* and *flats* had the highest proportional representation of all classes at most entropy levels (defined by values greater than or equal to 0.95, 0.90, 0.85, and 0.75, respectively). At the highest entropy level (0.99) the class proportions were more variable. This is important as both *backslopes* and *flats* showed dominant proportions of total area at different scales (*backslopes* at finer scales, and *flats* at coarser scales) within CRP Parcels. The presented approach allows an improved implementation of landform models by incorporating an uncertainty assessment and sensitivity analysis for a variation of spatial scales.

## I. INTRODUCTION

The magnitudes of many hydrological, geomorphologic, and biological processes active in the landscape are sensitive to topographic position [7]. Terrain attributes are the basis for identifying elementary landform units (in systems known as landform taxonomies), which in turn are important to the study of these processes. Although easy to conceptualize cartographically, common “crisp” models of landform units ignore inherent variations in the landscape and transitional states between two or more classes. This can result in fragmented or chaotic spatial patterns, limited flexibility to adjust for different types and scales of landforms, and thus a lack of generality across different landscapes [6]. Most current approaches of landform

classification are specific to a particular scale or a narrow range of scales and ignore scale effects [10]. Thus there exists a need to incorporate scale-specific variations of landform units in order to address the levels of uncertainty as has been demonstrated for e.g., fuzzy land cover classes [3].

Uncertainty in common spatial representations of landforms can be characterized in different ways: (a) Precise boundaries of crisp landform representations assume that all important change occurs at designated boundaries ignoring gradation and transitions in the landscape; (b) The range of surface measurements used to parameterize location, and thus the extent of landform objects and classes varies with changing scale; (c) Landforms are defined by inherently vague linguistic or semantic concepts such as “rolling”, “flat”, or “hilly” [6]. Consequently, the two forms of uncertainty in landform representations that can be identified for a specified scale are vagueness and ambiguity. Ambiguity implies that a single location may relate to different classes under varying classification schemes [4]. Vagueness refers to a lack of distinctness between ill-defined or fuzzy classes of objects or individual objects and often links to linguistic concepts [5]. Accounting for vagueness and ambiguity at a specified scale and across different scales is essential in maintaining the generality and applicability of landform taxonomies to various landscapes and paradigms.

This research examines the effect of scale-specific characteristics of geomorphometric classes on the resultant taxonomy using a fuzzy set approach. The presented model identifies proportions of geomorphometric classes and regions of highest classification confusion as well as measures of classification stability for each location across different scales. In a case study we demonstrate how this model could be used to improve reliability of soil and vegetation mapping at scales related to field or parcel-specific agronomic and natural resource management decisions. We use parcel data of the Conservation Reserve Program (CRP),

a voluntary cropland retirement program for the Delaware River Basin in Northeast Kansas using 10-meter resolution DEMs.

## II. BACKGROUND: SCALE AND UNCERTAINTY IN LANDFORM TAXONOMIES

Most existing geomorphometric systems are Boolean in nature. The resulting units contain maximum internal homogeneity and external heterogeneity - a paradigm referred to as the ‘double-crisp’ model, which is easy to conceptualize cartographically, but ignores ambiguity and imprecision as addressed earlier [1]. Modeling landforms without understanding the inherent uncertainty within each class can cause error propagation. Class-level uncertainty is well-studied in fields of remote sensing and land cover/land use, but has received limited attention in geomorphometry [2, 9]. The level of uncertainty may vary across landform classes that compose a single taxonomy. Some of the classes can be more crisp or more fuzzy or ambiguous than others depending on underlying terrain attributes or definitions used. This illustrates a need for creating generic techniques to quantify uncertainty for landform objects derived from DEMs.

One unresolved question in landform models is how to quantify and represent uncertainty across scales. The range of scales over which landforms are characterized represents a more theoretical, and essentially a geographical problem: information and relationships derived at one scale can change as the scale changes [10]. Two ways scale has been accounted for in landform taxonomy are the calculation of surface parameters used to define landform classes over a range of window sizes or changing the underlying DEM resolution [10]. How uncertainty in landform taxonomies varies between classes has been relatively ignored.

## III. METHODS

### A. Workflow

To account for scale, different window sizes ranging from the DEM resolution (10 m) to the maximum window size (100 – 200 m) were used to create a fuzzy set-based landform taxonomy. A simplified fuzzy landform system consisting of six classes was derived [8] (Fig. 2). We (1) calculated terrain attributes and converted them to fuzzy semantic constructs that characterize semantic landforms for each scale [6], (2) carried out fuzzy overlay to compute fuzzy surfaces of each landform class across different scales, and created defuzzified (crisp) landform classes, and (3) examined classification stability across scales based on different entropy levels.

### B. Terrain attributes and fuzzy semantic import

Landforms were semantically characterized based on specific terrain attributes that allowed the development of a suite of

quantitative characteristics composing each landform class. Land surface parameterization was carried out using Evan’s second-order polynomial method and surfaces of terrain attributes were calculated for multi-scale surface characterization with a combination of customized algorithms in Python, MATLAB, and R [10].

Fuzzy set theory overcomes weaknesses of crisp classifications by accounting for soft class boundaries due to inherent ambiguity and vagueness as parts of the landscape structure. Each location in the landscape can be a partial member to one or more landform classes indicated by continuous degrees of membership in the range [0,1], with 1 equal to a prototypical or full membership, and 0 equal to non-membership. Within this study, we utilize fuzzy semantic import (SI) models to convert terrain attributes to fuzzy set memberships on a continuous scale [0,1] in a defined fuzzy set [6]. The SI models were based on first-order polynomials as fuzzy membership functions (Fig. 1).

The membership functions were parameterized using existing definitions and statistical distributions of the terrain attributes over the study area. The resulting fuzzy sets were used to model semantic constructs for the different landforms; each location (pixel) was given a membership to each semantic construct of each terrain attribute for each scale (Table 1, Fig. 2).

### C. Fuzzy overlay and defuzzification

Fuzzy overlay of semantic constructs was performed to derive fuzzy surfaces of different landform classes using the fuzzy logic intersect operator (MIN operator). The theoretical

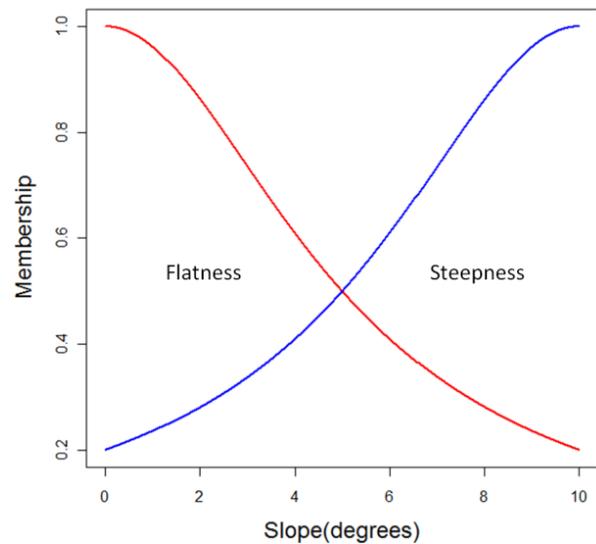


Fig. 1. Representation of a single terrain attribute (slope) with multiple semantic constructs using first-order polynomial semantic import functions.

TABLE 1. TERRAIN ATTRIBUTES AND THEIR SEMANTIC CONSTRUCTS

Terrain Attribute	Construct
Elevation Percentile	(1) Highness
	(2) Lowness
Slope	(3) Steepness
	(4) Flatness
Tangential Curvature	(5) Tan. Convexity
	(6) Tan. Concavity
	(7) Profile Convexity
Profile Curvature	(8) Profile Planarity
	(9) Profile Concavity
	(10) - Balanced
Relative Profile Curvature	(11) Balanced
	(12) +Balanced

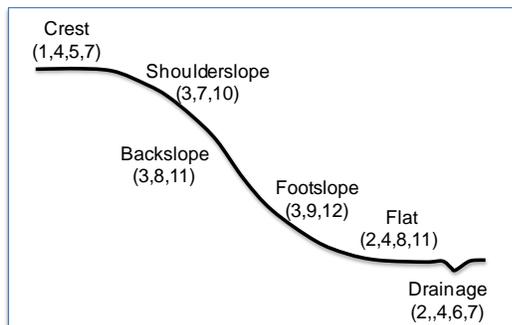


Fig. 2. Geomorphometric classes and their semantic constructs which are used to describe landforms.

basis for the application of this operator is the limiting factor principle of ecology [8]. Six fuzzy landform class layers were created for each scale (36 surfaces in total), with cell values indicating the degree of membership to a particular class. To derive a final, crisp layer of six landform classes, defuzzification was conducted. An overlay operation was applied to the six fuzzy landform layers utilizing the maximum of membership (MOM) method for each location (Fig. 3) [11]. At each location the landform class with the highest membership value was selected to define the crisp class; this procedure was repeated for each scale.

D. Analysis of stability and entropy

To determine the ambiguity and stability of specific landform classes across scales, a method incorporating the use of both crisp and fuzzy landforms was developed. First, the degree

of classification uncertainty (or confusion) was measured at each location for each scale using classification entropy, a derivation of the Shannon-Weiner Diversity Index [1]. Next, the proportions of MOM-based crisp landform classes that fell within specific ranges of entropy were determined.

A semantic landform class was considered ambiguous or unstable if high area proportions can be found at locations of high entropy. The ranges of entropy values utilized were  $\geq 0.75$ ,  $\geq 0.85$ ,  $\geq 0.9$ ,  $\geq 0.95$ , and  $\geq 0.99$ . Thus locations of each crisp landform class that spatially corresponded with entropy values equal to or greater than each threshold level were extracted (Fig. 4). This procedure was carried out for each scale.

IV. RESULTS

Within each scale, the crisp (defuzzified) classes *backslopes* and *flats* were the ones that showed the highest proportions of areas with entropy levels 0.75 – 0.95. At the entropy level 0.99 the class proportions were more variable (Figure 5). Over the entire study area, the proportion of *backslopes* gradually decreased with coarser scale, the one of *flats* increased, with inconsistent trends in the other four classes. This finding is in line with the conception of *backslopes* as the transition between uplands (*crests* and *shoulderslopes*) and lowlands (*footslopes* and *flats*).

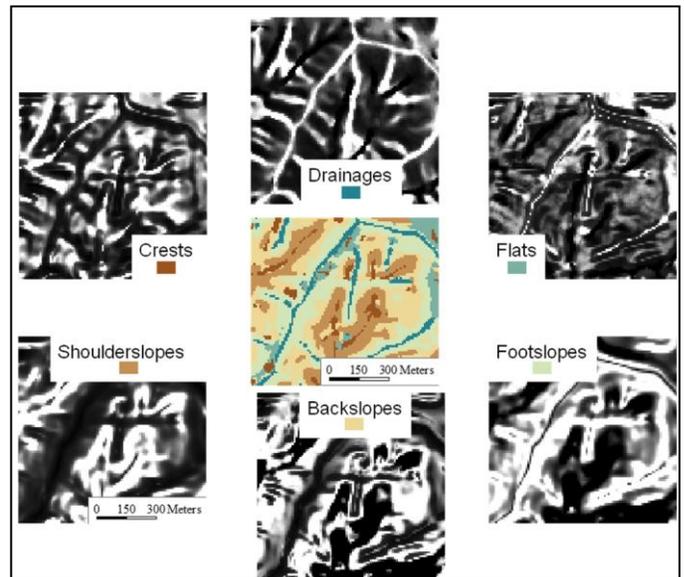


Fig. 3. Six fuzzy landform surfaces as input to the Maximum of Membership (MOM) defuzzification method to create a single layer of crisp landform classes.

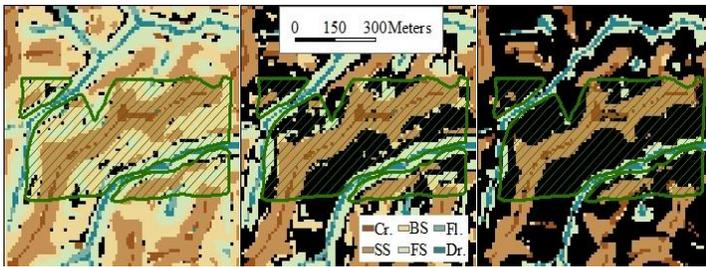


Fig. 4. Locations at different entropy levels (shown in black) at 0.95 (left), 0.85 (center), and 0.75 (right) underlain a CRP parcel (green hatched polygon).

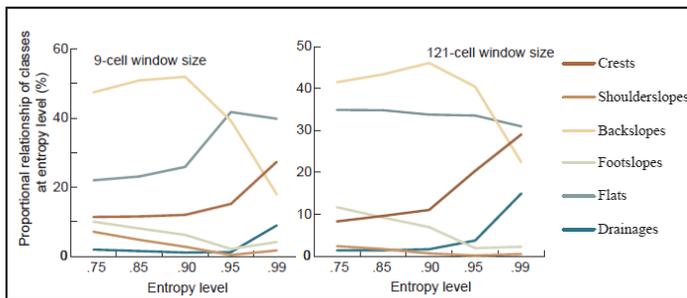


Fig. 5. Proportions of crisp (defuzzified) landform classes at varying levels of entropy and for different scales in the study area (shown for 9-cell and 121-cell window sizes).

This result is interesting since the total area of both classes - *backslopes* and *flats* - were dominant at different scales (*backslopes* at finer scales, and *flats* at coarser scales) within CRP parcels and in the entire study area. Spatial overlay of the resulting landform classes and related uncertainty by CRP units at different scales allowed the evaluation of individual parcels from a new perspective: in addition to landform proportions at each scale the proportions of unstable locations defined by entropy levels could be analyzed. For *backslopes* it could be shown that the class-specific area proportion was higher within CRP parcels than in the entire study area. Therefore, in the context of a specific land cover or land use (here CRP) there may be a higher potential for misclassification i.e., higher instability of large proportions within CRP units, based simply on the occurrence of classes and their frequency within it.

#### IV. DISCUSSION AND CONCLUSIONS

The presented consistent relationships (Fig. 5) can be construed in several ways: (1) The semantic classes *backslopes* and *flats* could truly be interpreted as transitional classes, in that *backslopes* act as the transition between *uplands* and *lowlands* area (sediment detachment and sediment deposition); *flats* act primarily as the transitional zone between *slopes* and *drainages*, the area of the highest level of hydrologic activity; (2) Areas of highest uncertainty (0.99-level entropy locations) appear to be less tied to specific classes but more to specific locations of high instability, due to high variation in the land surface. The

corresponding class proportions deviate from patterns shown at other levels of entropy, and are independent of scale; (3) The same locations at the highest level entropy may be also independent of the overall uncertainty of the classification system and more a result of artifacts from DEM processing or measurement [10]. However the fact that the same effect can be observed consistently across all scales casts some doubt on the latter point.

An important outcome from this study is the improvement of scale-specific landform mapping by incorporating uncertainty information. By accounting for inherent uncertainty i.e., vagueness and ambiguity using a fuzzy set approach, it was possible to determine that some classes in a geomorphometric hillslope model are naturally unstable independent of the scale of analysis. The exemplified application to CRP parcels showed that the high level of inherent uncertainty in crisp representations of some landform classes such as *backslopes* and *flats* could have serious consequences for decision-making regarding land retirement of erosion-susceptible units based on topographic position. The presented approach would thus have considerable potential for improving land management, conservation practices, or ecological modeling efforts.

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